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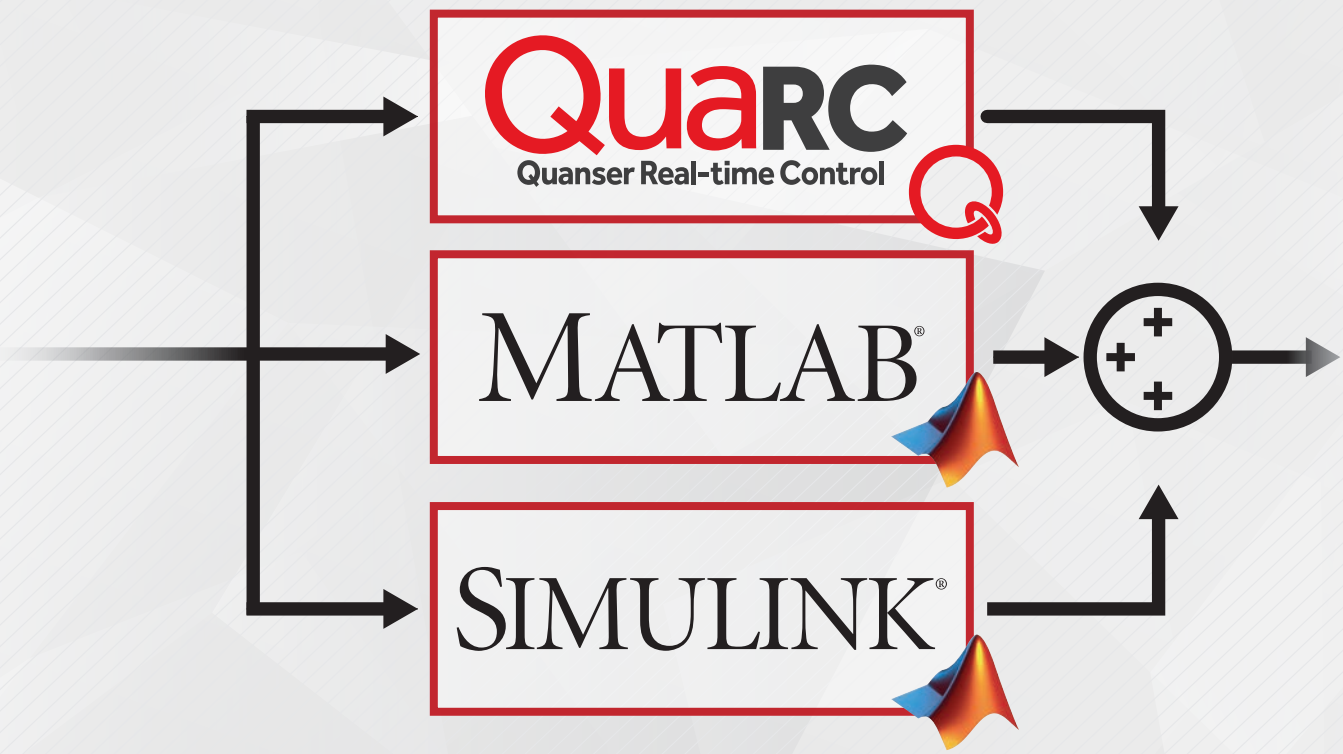

MATLAB/Simulink and QUARC Primer

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Contents

1 MATLAB

1.1 Introduction

MATLAB®is both a programming language and a software environment that integrates high-level computation, visualization and programming into an easy-to-use environment. At its core, MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. This allows MATLAB the capability to perform computationally intensive tasks faster than with traditional programming languages. At a basic level, MATLAB can be viewed as essentially a highly sophisticated calculator.

The MATLAB system consists of these main elements:

- **Desktop Tools and Development Environment**

  This part of MATLAB is the set of tools and facilities that are used to access and interface with the MATLAB functions and libraries. It includes: the MATLAB desktop and Command Window, an editor and debugger, a code analyzer, browsers for viewing help, the workspace, and other tools.

- **Mathematical Function Library**

  MATLAB includes an extensive library of computational algorithms ranging from elementary functions, like sum, sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms.

- **The Language**

  The MATLAB language is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features. Using the MATLAB language you can create both small quick-and-dirty scripts to automate specific tasks, and large-scale complex applications for repeated efficient reuse.

- **Graphics**

  MATLAB has extensive facilities for displaying vectors and matrices as graphs. In addition, it also includes high-level functions for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. There are also tools and facilities to fully customize the appearance of generated graphics as well as build complete graphical user interfaces for your MATLAB applications.

- **External Interfaces**

  The external interfaces library allows you to write C and Fortran programs that interact with MATLAB. It also includes facilities for calling routines from MATLAB (dynamic linking), and for reading and writing MAT-files.

1.2 The MATLAB Environment

1.2.1 Desktop

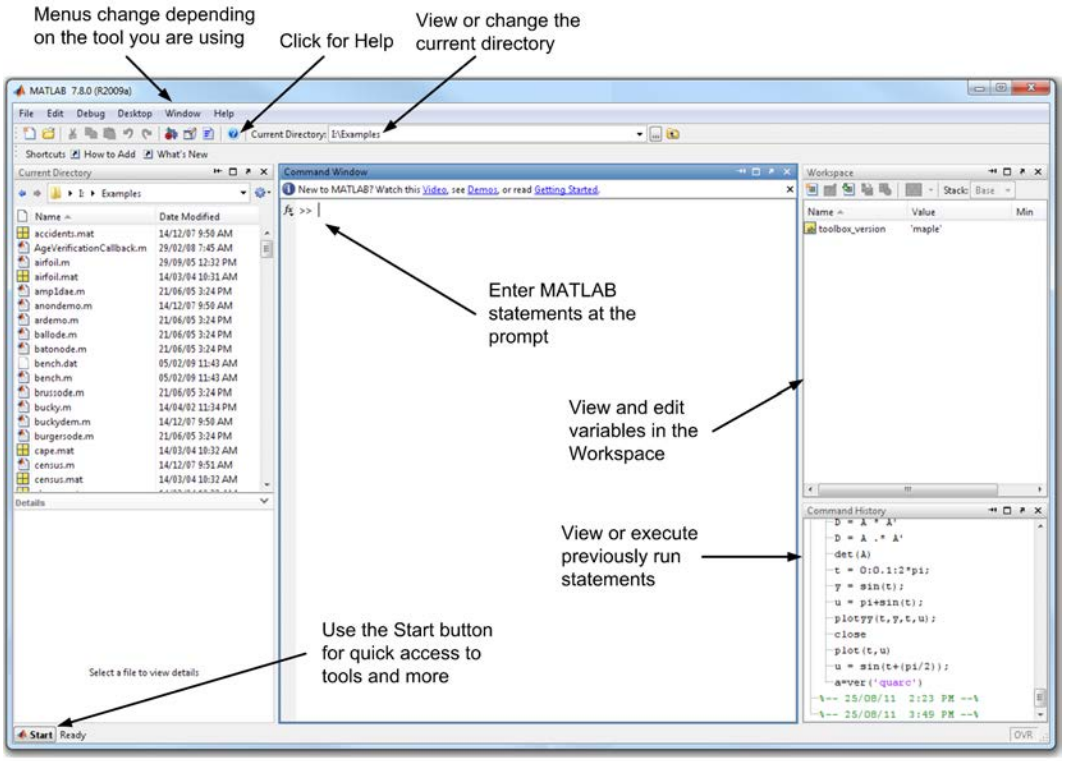


Figure 1.1: Basic elements of the MATLAB Desktop.

The desktop is the primary entry-point to the MATLAB environment. The desktop includes graphical tools that can be used to enter commands, configure the current working directory, view and manipulate workspace variables, view and execute previous statements, and launch additional programs and features. The elements of the desktop are shown in Figure 1.1, and a more detailed description of the basics of working on the desktop is included in Section 1.2.2.

1.2.2 Getting Started

To begin working with MATLAB, several aspects of the manner in which data is accessed and stored in MATLAB should be understood.

The fundamental unit of data in any MATLAB program is an array or matrix. An array is a collection of data values organized into rows and columns that can be represented by a single name. It is important to note that scalars values are also treated as arrays, that is, they are simply arrays with one row and one column. Arrays are discussed more in-depth in Section 1.2.5.

Within the MATLAB environment, variables are stored in the *Workspace*. The MATLAB workspace is somewhat akin to a visual representation of the memory used by a traditional program. Variables that are created on the workspace are accessible from any of the MATLAB tools, but must be saved for future use before MATLAB is closed.

The current working directory, which appears in the *Current Directory* tool on the desktop, is where data can be stored permanently for future use. The other location that is primarily accessed by MATLAB is the installation folder on the local machine where the functions that makeup the mathematical function library are stored. These function can be opened and manipulated to create custom derivations of existing MATLAB commands.

1.2.3 Functions and Commands

The command window shown in Figure 1.1 is the primary interface for working in the MATLAB environment. Commands are entered at the command prompt (») and executed when the user presses "Enter". When the command is complete, any appropriate output is displayed and the prompt reappears. If a function that returns a value or dataset is called and a user-defined variable is not specified, MATLAB automatically stores the answer in the variable *ans* in the workspace. The result of a command can be visually suppressed by including the ";" character after the command. For example, the following command will create a vector of numbers from 1 to 100:

```
» v = [1:100];
```

Expressions are generally commands that are called which use reserved keywords or operators. As with any programing language, there are several reserved keywords in MATLAB that cannot be overridden that are shown in Table 1. To see a complete list of the reserved keywords, enter the **iskeyword** command.

Keyword	Value
i and **j**	Imaginary
Inf	Infinite
NaN	Not-a-Number
realmin and **realmax**	Smallest and largest floating-point number
eps	Floating-point relative precision
pi	π (3.14159...)

Table 1: Reserved keywords

Functions on the other-hand, are generally small sub-programs that are stored in .m files and when called with the specified parameters, return a result. Functions are denoted by the brackets that follow their name "(..)" which hold the parameters that are passed to the function. MATLAB has a large library of functions that can be used for a wide range of mathematical operations and data analysis. An overview of creating custom functions is included in Section 1.4.2.

There are several rules that must be followed when creating variables:

1. No spacing within the name.
2. Case sensitive (ITEMS is not the same as ITems)
3. Can be a maximum of 31 characters.
4. Must start with a letter.
5. Punctuation is not allowed.

When changing the value of a variable, the new entry overwrites the original value. Keep in mind that if a variable is changed, calculations must then be re-executed as MATLAB uses the value it knows at the time the command is evaluated.

1.2.4 Operators

The following are the most common mathematical operators used to create MATLAB expressions:

Expression	Definition
A+B	Addition
A-B	Subtraction
A*B	Multiplication
A/B	Division
B^2	Exponent
A'	Complex conjugate transpose

The majority of the commands listed above can be used for both scalar operations and matrix operations. For example, 2 * 4 = 8 and at the same time A * B is the matrix multiplication of the matrices A and B. To perform array operations such as an element-wise array multiplication or division, include a "." before the operator. Perhaps the most important operator, however, is the %..% operator which denotes a single line comment.

1.2.5 Working with Arrays

The following commands are most commonly used to create, access and manipulate data arrays:

- **Creating Arrays:** The columns of an array are separated by a space or comma, and the rows by a semi-colon. For example,

```
» A = [1 2 3 4]
```

Outputs:

```
A =
     1 2 3 4
```

whereas the command

```
» A = [1; 2; 3; 4]
```

Outputs:

```
A =
    1
    2
    3
    4
```

Therefore to create a square matrix, you use a combination of the two

```
» A = [1 2 3; 4 5 6; 7 8 9]
```

Outputs:

```
A =
    1     2     3
    4     5     6
    7     8     9
```

- **Accessing Arrays:** In MATLAB, individual data values within an array can be accessed by including the name of the array followed by subscripts in parentheses that identify the row and column of the particular value of interest. For example, the second element (3) of a matrix A with three elements, A = [1 3 6]; can be accessed using the command A(1,2) where 1 represents the first row and 2 represents the second column).
- **The Colon Operator:** The colon operator is one of the most useful operators in MATLAB. It denotes a numerical range dependent on the values on either side of the operator. When accessing a range of elements in an array, the following definitions can be used:

Expression	Definition
A(:,j)	the jth column of A
A(i,:)	the ith row of A
A(:,:)	the equivalent two-dimensional array. For matrices this is the same as A.
A(j:k)	A(j), A(j+1),...,A(k)
A(:,j:k)	A(:,j), A(:,j+1),...,A(:,k)
A(:,:,k)	the kth page of three-dimensional array A.
A(i,j,k,:)	a vector in four-dimensional array A. The vector includes A(i,j,k,1), A(i,j,k,2), A(i,j,k,3), etc.
A(:)	all the elements of A, regarded as a single column.

The colon can also be used to create regularly spaced expressions as follows:

Expression	Definition
j:k	the same as [j,j+1,...,k] (empty if j > k)
j:i:k	the same as [j,j+i,j+2i, ...,k] (empty if i = 0, if i > 0 and j > k, or if i < 0 and j < k)

- **Concatenation:** *Concatenation* is the process of joining small matrices to make bigger ones. In fact, the process of creating an array covered earlier uses the concatenation operator "[..]" to create arrays out of smaller elements. In MATLAB, smaller arrays or portions of arrays can be concatenated together simply by including their variable names in the definition. For example, if you declare an array

```
» A = [1 2 3 4]
```

then the command

```
» B = [A; A]
```

Outputs:

```
B =
    1     2     3     4
    1     2     3     4
```

and the command

```
» C = [B(:,1) B(:,3)]
```

Outputs:

```
C =
    1     3
    1     3
```

- **Deleting Rows and Columns:** You can delete rows and columns from a matrix using the concatenation operator "[..]". For example, if you declare an array:

```
» A = [1 2 3; 4 5 6; 7 8 9]
```

Outputs:

```
A =
    1     2     3
    4     5     6
    7     8     9
```

then the command

```
» A(:,2) = []
```

Outputs:

```
A =
    1     3
    4     6
    7     9
```

- **zeros and ones:** MATLAB has built-in functions to easily create basic matrices. The *zeros* function can be used to create an array of 0's, and the *ones* function can be used to create an array of 1's. For example,

```
» Z = zeros(2,4)
```

Outputs:

```
Z =
    0     0     0     0
    0     0     0     0
```

and the command

```
» F = 5 * ones(3,3)
```

Outputs:

```
F =
    5     5     5
    5     5     5
```

1.2.6 Mathematics

A list of the most commonly used math functions is shown in Table 2.

abs(x): absolute value	**acosh(x):** inverse hyperbolic cosine	**exp(x):** e^x
cos(x): cosine	**asin(x):** inverse sine	**fix(x):** round toward zero
sin(x): sine	**asinh(x):** inverse hyperbolic sine	**floor(x):** round toward minus infinity
tan(x): tangent	**atan(x):** inverse tangent	**imag(x):** imaginary part of complex number
cosh(x): hyperbolic cosine	**Atanh(x):** inverse hyperbolic tangent	**real(x):** real part of complex number
sinh(x): hyperbolic sine	**angle(x):** phase angle	**log(x):** natural logarithm
tanh(x): hyperbolic tangent	**ceil(x):** round toward plus infinity	**log10(x):** log base 10
acos(x): inverse cosine	**conj(x):** complex conjugate	**sqrt(x):** square root

Table 2: Some common math functions.

In addition there are several built-in operators and functions for common linear algebra expressions. For example, if a matrix is defined as

```
» A = [ 2 4 6; 3 4 5; 1 2 3]
```

then using the transpose operator '

```
» T = A'
```

Output:

```
T=
    2     3     1
    4     4     2
    6     5     3
```

and to perform matrix multiplication you can use the "*" operator

```
» B = [1; 2; 3];

» C = A*B
```

Output:

```
C =
    28
    26
    14
```

The period "." can also be used to specify an elementwise operation such as

```
» D = A.*A'
```

Output:

```
D=
     4    12     6
    12    16    10
     6    10     9
```

and to find the determinant you can use

```
» det(A)
```

Output: `0`

There are also several other functions for finding the inverse, row echelon form, eigenvalues, etc.

1.2.7 Polynomials

For the majority of operations that are performed on a polynomial expression, the polynomial is represented as a vector of the coefficients in descending order of power. For example, the polynomial $x = s^3 + 3s^2 + 2s + 1$ would be entered as:

```
» x = [1 3 2 1];
```

Zeros must be entered into the expression as place-holders if the polynomial is missing coefficients. For example, $x = x^4 + 1$ would be entered as

```
» x = [1 0 0 0 1];
```

There are several functions that can be used to perform common tasks with polynomials. For example, to find the value of the polynomial $p(x) = 3x^2 + 2x + 1$ at x = 3, 4, and 5 you would use the function **polyval** as

```
»p = [3 2 1];

»polyval(p,[3 4 5])
```

Output:

```
ans =
    86     162     262
```

To find the roots of a polynomial, use the function **roots** as

```
» p = [4 8 2];

» roots(p)
```

Output:

```
ans =
    -1.7071
    -0.2929
```

To multiply two polynomials together, use convolution as denoted by the **conv** function. For example, to multiply $(s+2)(s^2+4s+8)$ enter

```
» conv([1 2],[1 4 8])
```

Output:

```
ans =
    1     6     16     16
```

Similarly, to divide polynomials use the **deconv** function which returns the remainder and the result. For example, to divide $z = s^3 + 6s^2 + 16s + 16$ by $y = s^2 + 4s + 8$ enter

```
» z = [1 6 16 16];

» y = [1 4 8];

» [div,R] = deconv(z,y)
```

Output:

```
div =
    1     2

R =
    0     0     0     0
```

1.2.8 Common Commands

- **help** [command] - Opens the help article on the command specified.
- **general** - Provides the user with a list of general purpose commands.
- **ops** - Provides a list of operators and special characters.
- **elfun** - Provides a list of the elementary math functions.
- **specfun** - Displays all of the specialized math functions.
- **clc** - Clears contents of Command Window
- **clear** - Clears contents of Workspace
- **whos** - Returns a list of all the variables and arrays in the current workspace
- **diary** [filename] - Keeps track of everything done in a MATLAB session. A copy of all input and most output is echoed in the diary file
- **clear** [variable] - Clears variable specified
- **what** - Lists MATLAB specific files in directory

1.3 Figures and Graphing

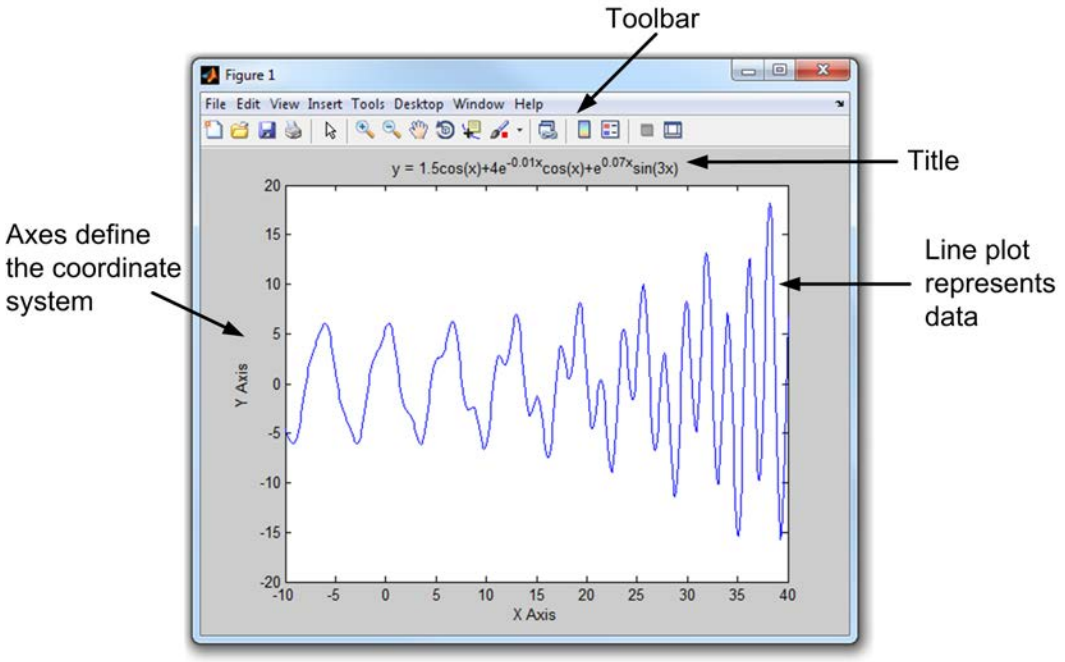


Figure 1.2: Basic elements of the Figure Window.

1.3.1 Figure Window

The MATLAB environment offers a variety of data plotting functions plus a set of Graphical User Interface (GUI) tools to create and modify graphics and plots. The figure window is used to display two and three dimensional plots, images, animations and GUIs. The window includes a wide range of plotting and analysis tools that can be used to customize figures and extract information about the data. The elements of the figure window are shown in Figure 1.2, and common tasks and graphing commands are outlined in Section 1.3.3.

1.3.2 Getting Started

There are different approaches that can be used to create and customize graphics and plots in MATLAB. The most common access-point is with the command **figure**. Once a figure window has been created, all subsequent commands that relate to graphics and plots are directed towards that window until a new figure is opened or the *CurrentFigure* property is updated. Though common practice is to create plots and graphics when a program is complete, figure windows can be created and updated during program execution to track changes in data graphically as the program executes. Some essential commands for graphing during program execution are included in Section A.

1.3.3 Creating 2-D Plots

The most commonly used functions for creating plots are listed below. In addition to these functions, several of the MATLAB toolboxes include specialized functions for creating topical plots for specific datasets. For example, the **bode** function which is detailed in 1.5 can be used to quickly create bode plots for system analysis.

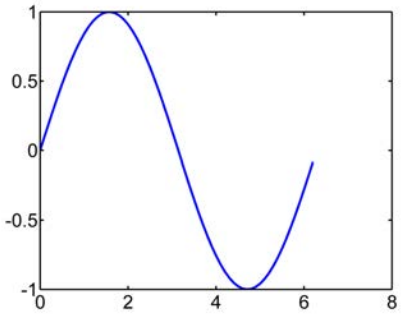
- **plot:** *plot* is the most common plotting tool. When you call *plot* with a single dataset, it creates a plot of the data with respect to the index of the data point. When you pass the function two arguments, the first dataset is plotted along the x-axis and the second along the y-axis. For example, if you create a dataset that holds some sinusoidal data points as

```
» t = 0:0.1:2*pi;

» y = sin(t);
```

then to create a plot of the resultant data

```
» plot(t,y);
```

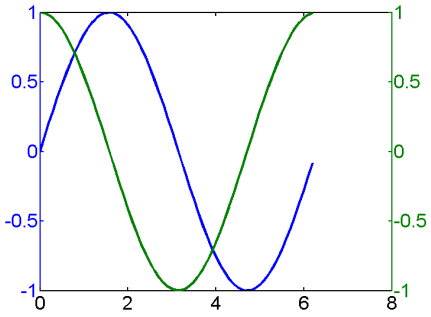
Output:



plotyy: *plotyy* is similar to the *plot* function, but it creates a plot of two datasets with their y-axis values on either side of the figure. For example, if you were to plot the previous dataset with an additional phase-shifted sinusoid you would enter

```
» u = sin(t+(pi/2));

» plotyy(t,y,t,u);
```

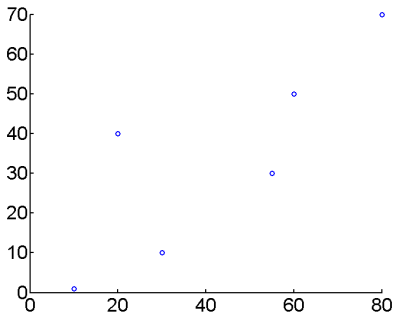
Output:



- **scatter:** *scatter* is used to create scatter plots. It can be called with two arguments corresponding to the x-axis and y-axis data respectively, or with additional properties to customize the type of icon and colour that appears at each point. For example, a scatter plot of the vector $x = [10, 30, 60, 20, 80, 55]$ and $y = [1, 10, 50, 40, 70, 30]$ would be created as

```
» x = [10,30,60,20,80,55];

» y = [1,10,50,40,70,30];

» scatter(x,y);
```

Output:



- **semilogx:** *semilogx* is similar to the plot function, but automatically creates logarithmic scale for the x-axis.
- **semilogy:** *semilogy* is similar to *semilogx*, but automatically creates logarithmic scale for the y-axis.

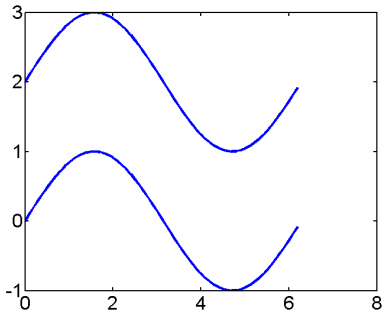
1.3.4 Managing Figures

Some essential commands for creating and customizing figures and plots are listed below. Some of these functions and commands can also be accessed through the figure window.

- **hold:** The *hold* command is used to tell MATLAB to graph subsequent plots in the current figure window. The property is turned on by calling *hold on*, and turned off by calling *hold off*. If the property is turned on, additional plots are automatically coloured differently to avoid confusion. To retain the current line and colour settings, the *hold all* command is used. For example, to graph the two sinusoids $y = sin(t) + {}^{\pi}/_{2}$ and $j = sin(t)$ you could enter

```
» t = 0:0.1:2*pi();

» y = sin(t)+2;

» j = sin(t);

» plot(t,y);

» hold on

» plot(t,j);
```

Output:



- **axis:** The *axis* command is used to set the scaling and appearance of the axes on the current plot. The most common syntax is *axis([xmin xmax ymin ymax])* where the min and max values correspond to the desired axes boundaries.
- **xlabel/ylabel/zlabel:** The *label* commands are used to set the title of the specified axis. For example, the command *xlabel('Time (s)');* will create or modify the title of the x-axis to "Time (s)".
- **title:** The *title* command is used to create or modify the title of the plot.
- **clf:** Calling the *clf* command clears the current figure.
- **close all:** Calling the *close all* command closes all figure windows.

1.4 Programming

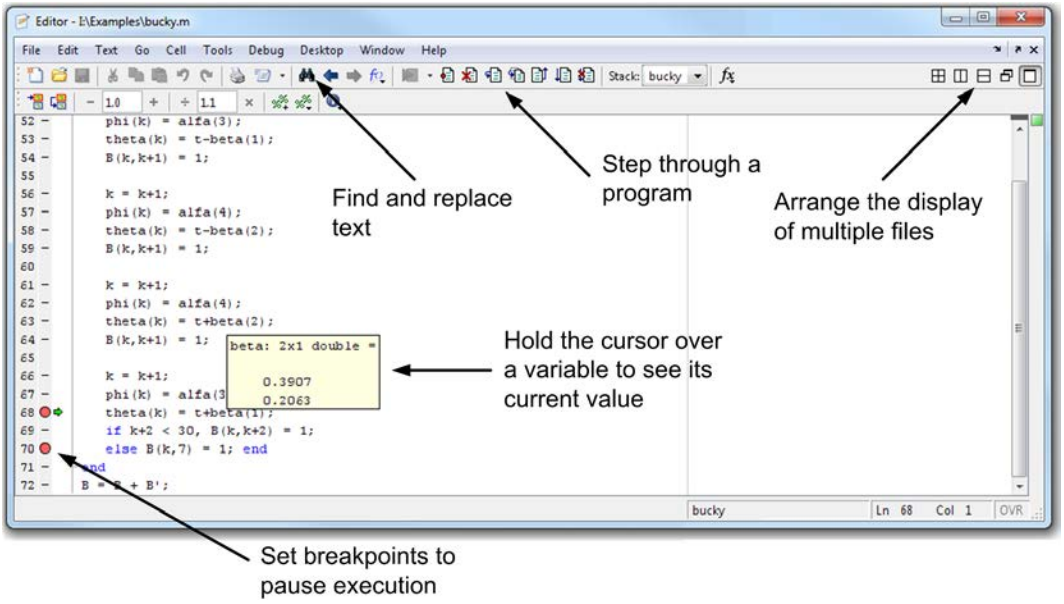


Figure 1.3: Basic elements of the Editor.

The Editor is essentially a development tool that can be used to create and edit applications in the MATLAB environment. The primary features of the Editor are an extensive debugging tool that allows you to step through your code as it executes, and a code analyzer that can help identify problems and potential improvements. The elements of the editor are shown in Figure 1.3.

1.4.1 Getting Started

The MATLAB environment is built using a high-level language that includes its own internal classes (data types), functions, object-oriented programming capabilities and the ability to import and interface with external technologies and applications. For most users, however, the programming tools and functions available in MATLAB can be viewed as a means to automate and reuse long sequences of commands and expressions. Programs written in MATLAB are saved as .m files and can be executed from the Desktop, or from the Editor.

The two main approaches to creating programs in MATLAB are scripts and functions. Scripts are essentially sequential sets of MATLAB commands written using the editor and saved as a .m file. In other words, a sequence of commands in the Command Window could be copied into the Editor window to create a script to accomplish the same result, and vise-versa. Users can also crate custom functions that behave in much the same way as the built-in MATLAB library functions. Approaches to creating functions are described in more detail in section 1.4.2. In addition, internal MATLAB functions can be viewed and edited using the Editor to create custom derivations of the built-in library functions.

In addition to the library commands and functions, MATLAB also includes conventional structures and program control statements. These statements control how the program is executed and include loops and conditions. These structures are discussed in more detail in 1.4.3.

1.4.2 User-Defined Functions

The fundamental difference between functions and scripts is that functions accept and return parameters when they are called, whereas scripts simply execute their command sequence. To create and run a script you simply write the program in the editor, and save the .m file in the current directory. You can then type in the name of the script at the command prompt, or click on the *Run* button on the editor menubar.

Functions on the other hand are denoted by the command *function* and are declared as follows:

```
function [return_parameters] = function_name(input_parameters)
    %Code goes here...
end
```

Variables that are passed into a function as arguments are accessed using the declarations in the prototype (function command). Parameters that are returned from a function hold the value of the return variable when the function execution is complete. For example, the following function returns the root-mean-square (quadratic mean) of a set of numbers:

```
function [rms] = calculate(values)

    rms = sqrt(sum(values.^2)/length(values));

end
```

The following rules apply to user-defined functions:

- The first few lines of the function program should be comments for clarification purposes.
- The only information returned from the function is contained in the output argument(s).
- Variable names can be used in both a function and the program that references it.
- When a function that returns more than one variable is called, all return parameters must be specified or it will only return only the fist parameter. For example, the function

  ```
  function [dist,vel,accel] = motion(x)

  ...

  end
  ```

  returns three parameters, but when called using the command

  ```
  » result = motion(x);
  ```

  it will only return the first value. For all three return parameters it must be called as

```
» [var1, var2, var3] = motion(x);
```

1.4.3 Program Control

The two most important elements of program control are *conditional* statements and *loops*. The two types of conditional statements are *if* statements and *switch* statements. The two types of loops are *for* statements and *while* statements. The elements of each are listed below:

- **if, else, and elseif:** The *if* statement (which may include *else* or *elseif*) enables you to select at run-time which block of code is executed. The selection of the particular code block is made depending on the condition specified in the statement using the relational operators listed in Table 3 and the logical operators listed in Table 4. Execution of the condition statement

  ```
  if (logical_expression)
      statements
  end
  ```

  can therefore be described verbally as:

  ```
  if condition_1 is (greater than/less than/equal to) ...  condition_2 (and/or)
  ...  then
      % Execute this code block
  (else/elseif condition_3)
      % Execute this code block
  end
  ```

  For example, to check if a variable is positive you might write

  ```
  if (num >= 0)
      positive = true;
  else
      positive = false;
  end
  ```

  The else and elseif statements further conditionalize the if statement. The conditional statements execute sequentially, and therefore *elseif* and *else* statements execute only if previous conditional statements were false. For example, the code sequence

```
val = 1;

if (val > 0)
    val2 = val + 1
elseif (val < 10)
    val2 = val + 2;
else
    val2 = val + 3;
end
```

sets *val2* to 2 because when the first condition is true, the other conditions are ignored even though they are also true. Conditional statements can also be nested within one another to handle multiple conditions and sub-conditions.

Operator	Description
<	Less than
<=	Less than or equal to
>	Greater than
>=	Greater than or equal to
==	Equal to
=	Not equal to

Table 3: Conditional operators.

Operator	Description
A & B	and(A,B) returns true if both conditions are true, otherwise false
A | B	or(A,B) returns true if one or all of the conditions are true, otherwise false
A	not(A) returns true if A is false, otherwise false

Table 4: Logical operators.

- **switch:** If you have a large set of conditions that relate to a single variable or expression, a *switch* statement is a more efficient method of execution control than a large *if...elseif* statement. The basic form of a switch is

```
switch expression (scalar_or_string)
    case value1
        statements % Executes if expression is value1
    case value2
        statements % Executes if expression is value2
    ...
        otherwise
    statements % Executes if expression does not match any case
end
```

The code blocks that follow each *case* statement execute only if the variable following the *switch* statement matches the condition in the *case* statement. The otherwise group executes if the value in the *switch* statement does not match any of the *case* conditions. For example, the following statement

```
switch num
    case -1
        val = 'negative one';
    case 0
        val = 'zero';
    case 1
        val = 'positive one';
    otherwise
        val = 'other value';
end
```

will compare the value of *num* to the values -1, 0, and 1 and set *val* to the appropriate string.

- **for:** The *for* loop continues to executes a statement or group of statements a predetermined number of times. Its basic form is

```
for index = start:increment:end
    statements
end
```

The default increment is 1. You can specify any increment, including a negative one. Execution terminates when the value of the index is **greater than** or **less than** the end value for positive and negative increments respectively. For example, this loop executes five times

```
for n = 2:6
    x(n) = 2 * x(n - 1);
end
```

and if x = 1; when the loop begins sets the value of x to

```
x =
    1     2     4     8    16    32
```

- **while:** The *while* loop executes a statement or group of statements repeatedly as long as the controlling expression is true (1). Its syntax is

```
while (expression)
    statements
end
```

For example, this while loop finds the first integer n for which n! (n factorial) is a 100-digit number (1×10^{100})

```
n = 1;

while(prod(1:n) < 1e100)
    n = n + 1;
end
```

Exit a while loop at any time using the **break** statement.

1.5 Control Systems Toolbox

The Control System Toolbox is a set of functions written in the MATLAB language that make it convenient to build the system models and perform the analyses that are used in control systems engineering. A list of the most common commands for systems analysis, modeling and control are listed in section Section 1.5.1.

1.5.1 Common Commands

- **sys=tf(num,den):** Given numerator and denominator polynomials, *tf* creates the system model as a transfer function object. The continuous-time transfer function is created with numerators (*num*) and denominators (*den*). The transfer function (TF) object can then be used in conjunction with several additional analysis tools.
- **T=feedback(G,H):** Given the models of two systems as TF objects, (G,H), *feedback* returns the model of the closed-loop system. Negative feedback is assumed, but an optional argument can be used to handle the positive feedback case.
- **y=step(T):** Given a continuous system, *step* returns the response to a unit step input. If *step* is called without output arguments, the function creates a plot of the response.
- **y=impulse(T):** Given a continuous system, *impulse* returns the response to a unit step input. If *step* is called without output arguments, the function creates a plot of the response.
- **output=lsim(sys,u,t):** Given a continuous LTI system, a vector of input values (u), a vector of time points (t) and a set of initial conditions, *lsim* returns the time response. *lsim* plots the time response of the LTI model sys to the input signal described by [u,t]. The time vector (t), consists of regularly spaced time samples and the input (u) is a matrix with as many columns as inputs whose i^{th} row specifies the input value at time t.
- **damp(T):** Given an LTI system, *damp* returns the closed loop system poles, their damping ratios and their natural frequencies. The output is three columns in the order of poles, damping ratios and natural frequencies respectively.
- **rlocus(F):** Given a transfer function (F(s)) of an open-loop system, *rlocus* produces a root locus plot that shows the locations of the closed-loop poles in the s-plane as the loop gain varies from 0 to ∞.
- **[gainsk,polesk]=rlocfind(F(s)):** Given a transfer function (F(s)) from the characteristic equation, *rlocfind* allows the user to select any point on the locus with the mouse and returns the value of the loop gain that will make that point be a closed-loop pole (gainsk). It also returns the values of all the closed-loop poles for that gain value (polesk).
- **bode(G,w):** This commands generates a Bode plot of the system (G) over the frequency range (w) in rad/s. The magnitude $|G(j\omega)|$ is plotted in decibels (dB), and the phase in degrees. If called with the return arguments [mag,phase], this command returns the magnitude in the column vector [*mag*], and the phase angles in degrees in the column vector [*phase*].
- **w = logspace(a,b,n):** Generates frequency values that are uniformly spaced on the logarithmic scale. It returns a row vector containing *n* points from *a* to *b* that are uniformly spaced on the logarithmic scale.
- **margin(G):** Generates a bode plot with the margins and crossover frequencies indicated.
- **[gm,pm, p, g] = margin(G):** Generates the output variables, gain margin (gm), phase margin (pm), phase crossover frequency (p), and gain crossover frequency (g).

1.6 Example 1 - Toy Train

Problem Description: The toy train shown in Figure 1.4 consists of an engine and a car joined together by a spring. F represents the force applied by the engine, the spring has a stiffness coefficient of k, and the mass of the engine and the car are represented by m_1 and m_2 respectively. The effects of friction cannot be neglected. To create a model of the system in MATLAB, we will begin by drawing the Free Body Diagram (FBD) of the engine and the car, assuming that the train only travels in one direction.

Note: The rolling friction of the train is $F_f = \mu m g \dot{x}$, where μ represents the coefficient of rolling friction. In the vertical direction, the gravitational force is canceled by the normal force applied by the ground. Consequently, there will be no acceleration in the vertical direction.

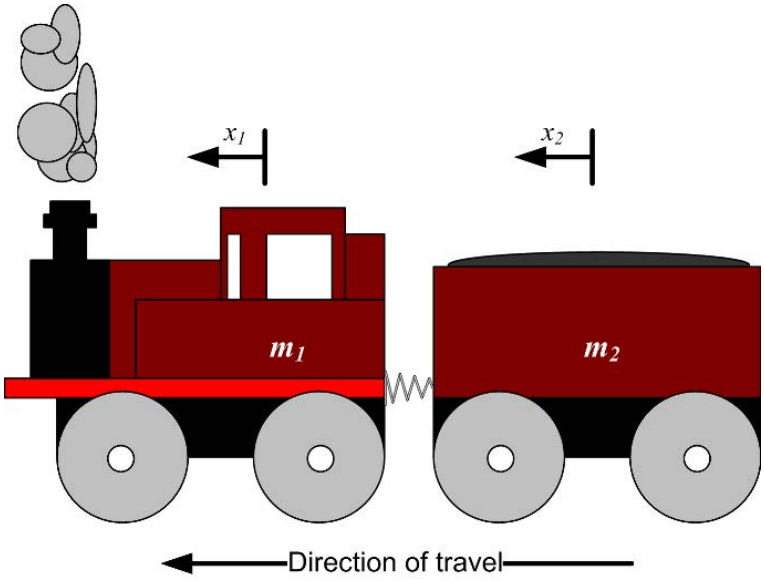


Figure 1.4: Toy train.

Next, we will derive the equations of motion for the system. Using Hooke's Law, we can relate the forces applied to the engine and the car to the change in their positions shown in Figure 1.4:

$$F_1 = k(x_1 - x_2)$$
$$F_2 = -k(x_1 - x_2)$$

Using the definition of rolling friction for the two bodies:

$$F_{f1} = \mu m_1 g \dot{x_1}$$
$$F_{f2} = \mu m_2 g \dot{x_2}$$

we can apply Newton's Second Law to relate the applied forces to the position of the cars:

$$m_1 \ddot{x_1} = F - k(x_1 - x_2) - \mu m_1 g \, \dot{x_1} \tag{1.1}$$
$$m_2 \ddot{x_2} = k(x_1 - x_2) - \mu m_2 g \, \dot{x_2} \tag{1.2}$$

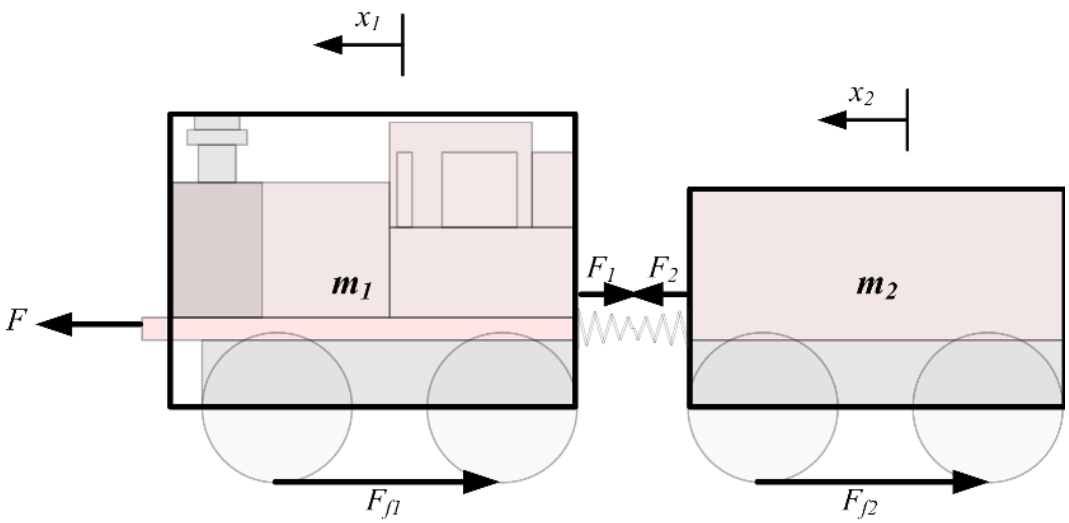


Figure 1.5: Free body diagram of the toy train

Next, we will find the Laplace transformations of Equation 1.1 and Equation 1.2. Recall that the transfer function of a system is the ratio of the Laplace transforms of its output and the input, assuming zero initial conditions.

$$\mathcal{L}\left[\dot{f}\right] = sF(s) - f(0)$$
$$\mathcal{L}\left[\ddot{f}\right] = s^2F(s) - sf(0) - \dot{f}(0)$$

Taking the Laplace Transform of Equation 1.1 and Equation 1.2 therefore yields:

$$m_1(s^2X_1(s) - sx_1(0) - \dot{x_1}(0)) = F(s) - k(X_1(s) - X_2(s)) - \mu m_1 g s X_1(s) \tag{1.3}$$
$$m_2(s^2X_2(s) - sx_2(0) - \dot{x_2}(0)) = k(X_1(s) - X_2(s)) - \mu m_2 g s X_2(s) \tag{1.4}$$

Since we are assuming the initial conditions are zero, if we let $\mu m_1 g = c_1$ and $\mu m_2 g = c_2$

$$m_1 s^2 X_1(s) = F(s) - k(X_1(s) - X_2(s)) - c_1 s X_1(s) \tag{1.5}$$
$$m_2 s^2 X_2(s) = k(X_1(s) - X_2(s)) - c_2 s X_2(s) \tag{1.6}$$

Recall that for the transfer function, we need the output over the input, where the input is $F(s)$ and the output is $X(s)$. Therefore, we must rearrange Equations 1.5 and 1.6 to achieve $X(s)/F(s)$.

from 1.5

$$X_1(s)(m_1 s^2 + c_1 s + k) = F(s) + kX_2(s) \tag{1.7}$$

from 1.6

$$X_2(s)(m_2 s^2 + c_2 s + k) = kX_1(s)$$
$$X_2(s) = \frac{kX_1(s)}{(m_2 s^2 + c_2 s + k)} \tag{1.8}$$

sub Equation 1.8 into Equation 1.7 and rearranging yields

$$\begin{aligned} X_1(s)(m_1s_2 + c_1s + k) &= F(s) + k\left(\frac{kX_1(s)}{m_2s_2 + c_2s + k}\right) \\ X_1(s)\left(m_1s_2 + c_1s + k - \frac{k^2}{m_2s^2 + c_2s + k}\right) &= F(s) \\ \frac{X_1(s)}{F(s)} &= \frac{1}{m_1s_2 + c_1s + k - \frac{k^2}{(m_2s^2+c_2s+k)}} \end{aligned}$$

or

$$\frac{X_1(s)}{F(s)} = \frac{m_2s^2 + c_2s + k}{m_1m_2s^4 + (m_1c_2 + m_2c_1)s^3 + (m_1k + c_1c_2 + m_2k)s^2 + (c_1k + c_2k)s} \tag{1.9}$$

Exercise 1: Create a system model in MATLAB

Using the following data:

m_1 = 1 kg
m_2 = 0.5 kg
k = 1 N/sec
F = 1 N
μ = 0.4

we can now create the system model. MATLAB is limited in its handling of symbolic variables, so it is essential that variables have declared numerical values.

```
» m1 = 1;
» m2 = 0.5;
» k = 1;
» F = 1;
» mu=0.4;
» c1=3.924;
» c2=1.962;
» num = [m2 c2 k];
» den = [m1*m2 (m1*c2+m2*c1) (m1*k+c1*c2+ m2*k) (c1*k+c2*k) 0];
» sys = tf(num,den);
```

The zeros are the roots of the numerator of the transfer function, and the poles are the roots of the characteristic equation. Therefore, to find the system poles and zeros you would use the following commands:

»roots(num)

Outputs:

-3.3219

-0.6021

»roots(den)

Outputs:

0

-3.9240

-2.8837

-1.0403

or

»zpk(sys)

Outputs:

Zero/pole/gain:

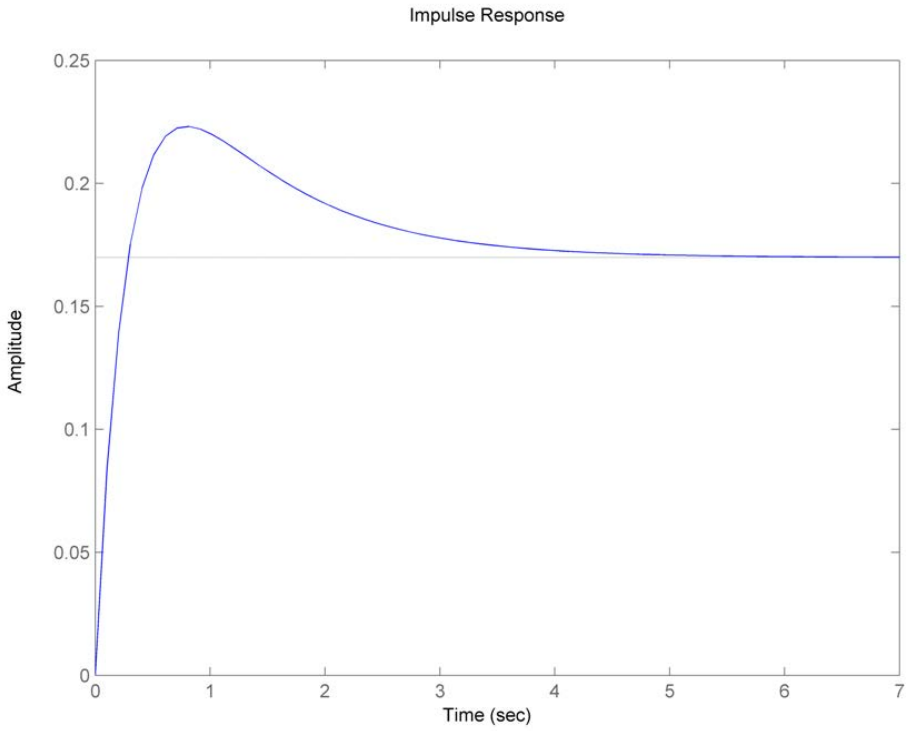
$$\frac{(s+3.322)\ (s+0.6021)}{s\ (s+3.924)\ (s+2.884)\ (s+1.04)}$$

Exercise 2: Plot the system response

To plot the impulse response of the open-loop system you could use:

»impulse(sys);

Output:

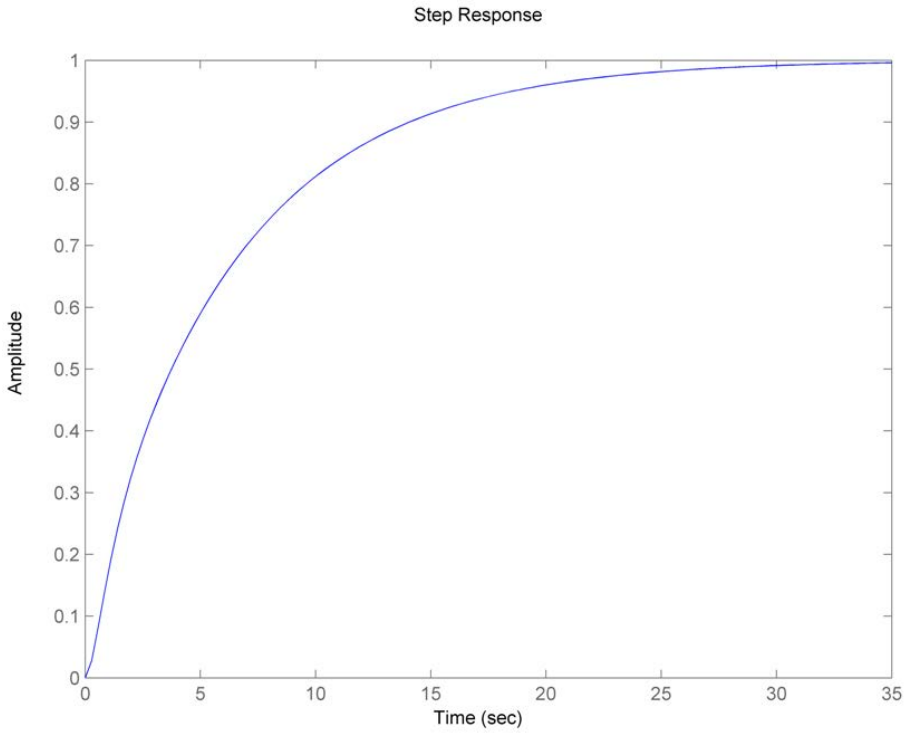


To plot the closed-loop step response of the system, assuming negative feedback and a unity gain on the feedback loop:

»cloop = feedback(sys,1);

»step(cloop);

Output:



2 Simulink

2.1 Introduction

Simulink is a software package for MATLAB that models, simulates and analyzes dynamic systems. In essence, the software provides a GUI for building models as block diagrams. A comprehensive library of the block elements that are used to build models is included, with tools to create or import custom blocks. MATLAB and Simulink are very closely integrated to facilitate simulation, analysis, and model revision in either environment at any time.

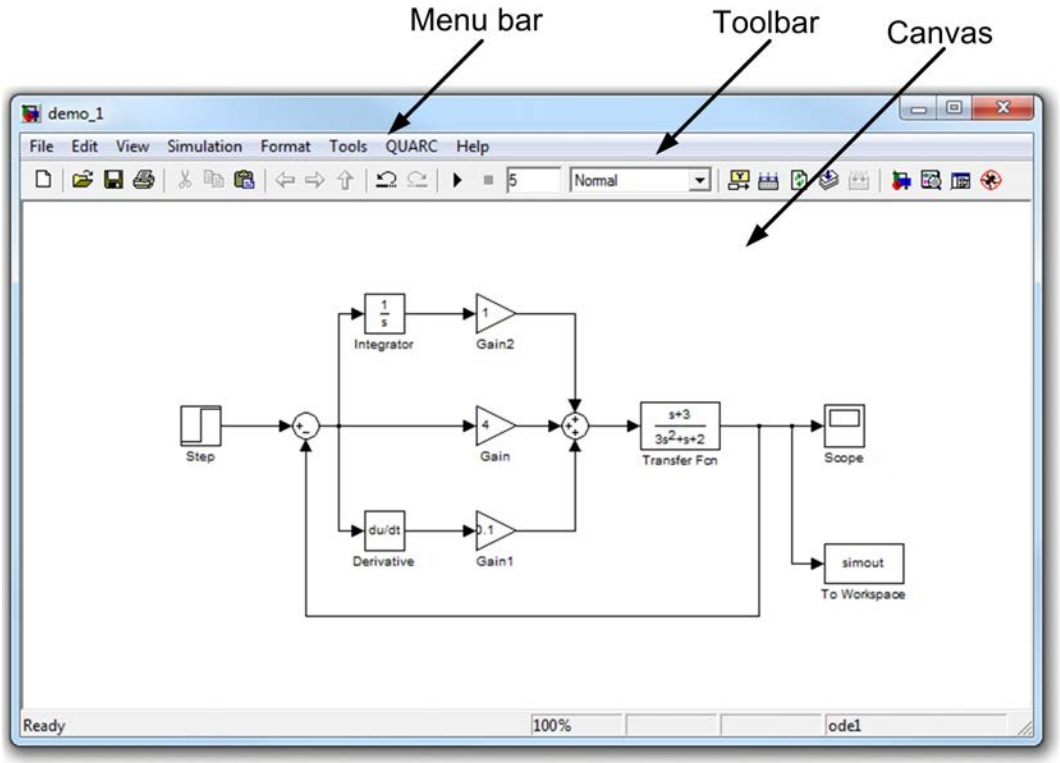


Figure 2.6: Basic elements of the Model Editor

2.2 The Simulink Environment

The Simulink software is divided into two elements: the *Library Browser* and the *Model Editor*. The library browser displays a tree-structured view of the block libraries that are installed. Models are created by copying blocks from the library browser into the model editor. The model editor provides a space, or canvas, for viewing and building models. The library browser and model editor are shown in Figure 2.7 and Figure 2.6.

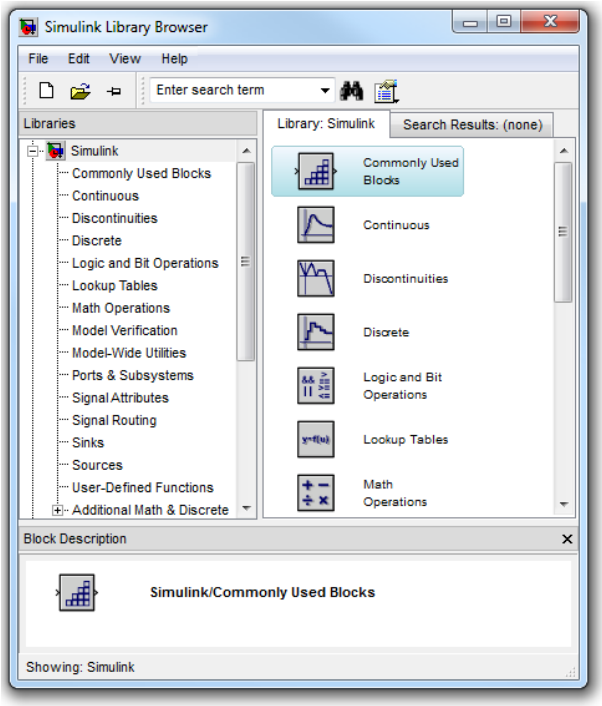


Figure 2.7: The Simulink Library Browser

2.2.1 Getting Started

Simulink models consist of three basic elements:

1. Sources
2. System Blocks
3. Sinks

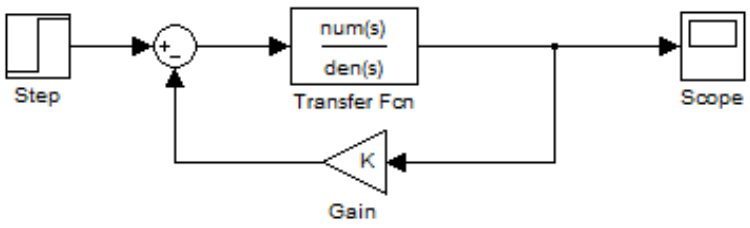


Figure 2.8: A Simulink model

The central element, the system, is the Simulink representation of a block diagram of the dynamic system being modeled. The sources are the inputs to the system (e.g. constants, signals generated using function generators, etc.) and the output of the system are received by sinks (e.g. oscilloscopes, output files, etc.). A system model is build by copying blocks out of the library

browser and into the model editor. They are then connected together by drawing connectors between the blocks. More details on creating system models are listed in Section 2.3, and a sample model is shown in Figure 2.8.

Once the model is complete, a basic simulation can be run by clicking on the *Start Simulation* button on the toolbar. The simulation will run for the amount of time listed next to the simulation controls, using the simulation model listed in the adjacent combo-box. The progress of the simulation appears in a progress bar at the bottom of the window. A more detailed account of configuring and running a simulation is presented in Section 2.4.

2.3 Building a Model

2.3.1 Sources

A good approach to creating a system model is often to begin by adding the sources for the model. The *Sources* blockset in the Simulink library contains a number of different sources including step inputs, random signals, sine waves etc. Some common sources are listed below:

- **Sine Wave:** The *Sine Wave* block outputs a sine wave with the specified Amplitude, Frequency, Phase, and Bias. The output signal is therefore
  $O(t) = Amp * Sin(Freq * t + Phase) + Bias$
- **Step:** The *Step* block outputs a step from the specified Initial Value to the Final Value at the appropriate Step Time.
- **Ramp:** The *Ramp* block outputs a ramp signal starting at the specified Start Time, with the appropriate Slope.
- **Pulse Generator:** The *Pulse Generator* creates pulses with the specified Amplitude, Period, and Pulse Width.
- **Signal Generator:** The *Signal Generator* creates various wave forms (Sine, Square, Sawtooth, etc.) with the specified Amplitude and Frequency.
- **From Workspace:** The *From Workspace* block, facilitates a custom input signal from the MATLAB workspace. The input (Data) must be in the form of a MATLAB matrix, using variables currently defined in the MATLAB workspace. The first column of the input matrix is the independent variable that corresponds to simulation time and must be monotonically increasing. The subsequent columns are values of the dependent variables corresponding to the independent variable in the first column [T,U(t)]. The block will produce as many outputs as there are dependent variables.
- **Unit Impulse:** There is no built-in block for creating a unit impulse, but it can easily be created by combining two *Step* blocks using a *Sum* block. If the two step signals are offset by the desired impulse width, and one subtracted from the other, the resultant output will crate an impulse of the desired magnitude. For example, the blocks shown in Figure 2.9 both have a magnitude of 1, but a difference in their *Start time* of 0.01s. The output will therefore be a impulse of 1 for 0.01s.

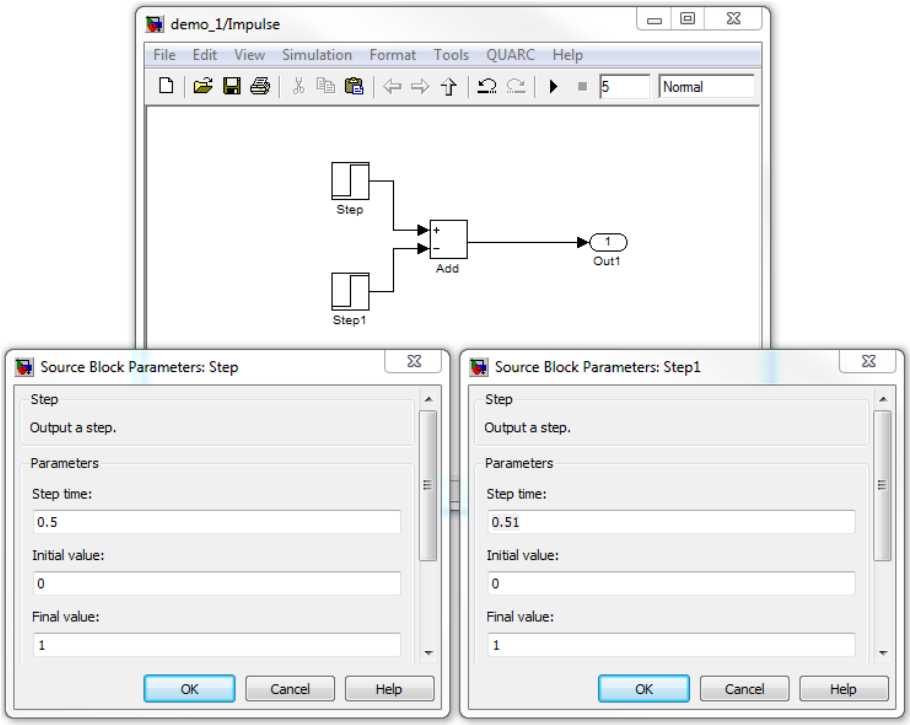


Figure 2.9: Impulse Signal

2.3.2 Building a System Model

Once the sources for the model are in place, it is time to assemble the blocks that make-up the dynamic model of the system. The most common libraries and block sets that are used to build system models are listed below.

- **Commonly Used Blocks:** The *Commonly Used Blocks* library includes the most commonly used elements of other libraries. It is always a good place to start when searching for a particular block or function. Most blocks in the library are straight-forward (Gain, Constant, Integrator, etc.) but others are a little more complicated and are outlined below:
  - **Mux/Demux:** The *Mux* and *Demux* multiplex scalar or vector signals. For example, to split a feedback signal to feed into separate elements of a controller, a *Demux* could be used. To combine the separate signals together into a 2-dimensional vector to pass to a *Scope*, a *Mux* could be utilized.
  - **Subsystem:** The *Subsystem* block contains a separate model (or models) as a sub-element of the overall model. The contents of the subsystem are treated in the same manner as the overall model, with data passed in and out of the system using input and output blocks.
  - **In1/Out1:** The *In1* and *Out1* blocks are used as an input port for subsystems or models.
  - **Data Type Convert:** The *Data Type Convert* block is a quantization block that converts "real-world" values input into the block to the proper data-type and scaling of the output. Normally, the properties of the output data type are automatically configured through inheritance.

- **Continuous:** The *Continuous* library provides a set of blocks to create continuous time system models. These blocks include basic elements such as a derivative, integrator, and delays. The library also includes complete system model blocks including a state-space, transfer function and zero-pole block. The system model blocks can be treated in the same way as the MATLAB equivalents, with the appropriate parameters input as polynomial vectors.
- **Discontinuities:** The *Discontinuities* library includes several blocks for describing discontinuous or non-linear system elements including friction, backlash, saturation and dead-zones.
- **Discrete:** The *Discrete* library provides many of the same elements as the *Continuous* library, but in discrete (sampled) time. The library also includes several filters and zero/first-order holds.
- **Math Operations:** The *Math Operations* library includes operators for performing common mathematical functions including addition, subtraction, products, trigonometry, polynomials, and exponents. The library also includes equivalent operations for signals including summation and gain blocks.

2.3.3 Sinks

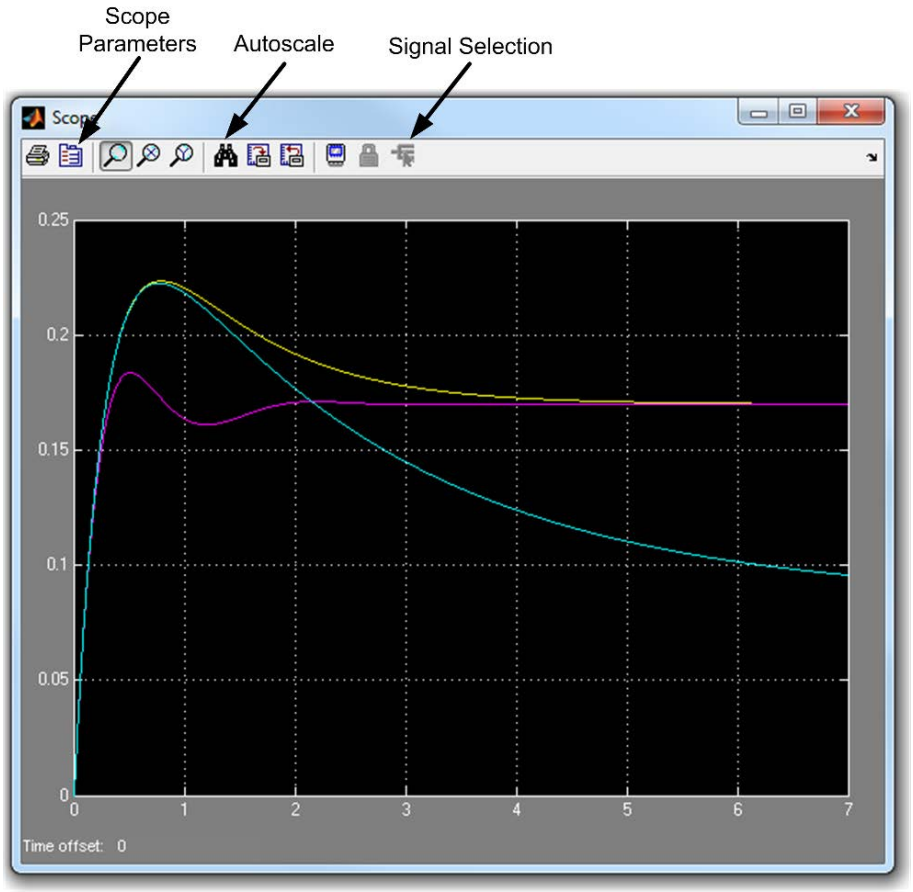


Figure 2.10: Scope Viewer

Sinks provide the means to store and/or view model data. The most common approaches to viewing and storing data are to either send the data to the MATLAB workspace using the *To Workspace* block, or using a *Scope*.

The scope block emulates an oscilloscope thereby producing plots of the input data. The *Scope Parameters* dialog allows users to customize the axis properties, sampling, and data history settings.

- The horizontal and vertical axis ranges of the scope can be set to any desired values.
- The vertical axis displays the actual value of the signal, and the horizontal axis represents time. If the time range is set to 'auto', the range will be the same as the simulation duration.
- The axes can also be auto adjusted to match the magnitude of the signal by clicking on the *Auto Scale* button on the toolbar (Binoculars).

2.4 Simulating a System Model

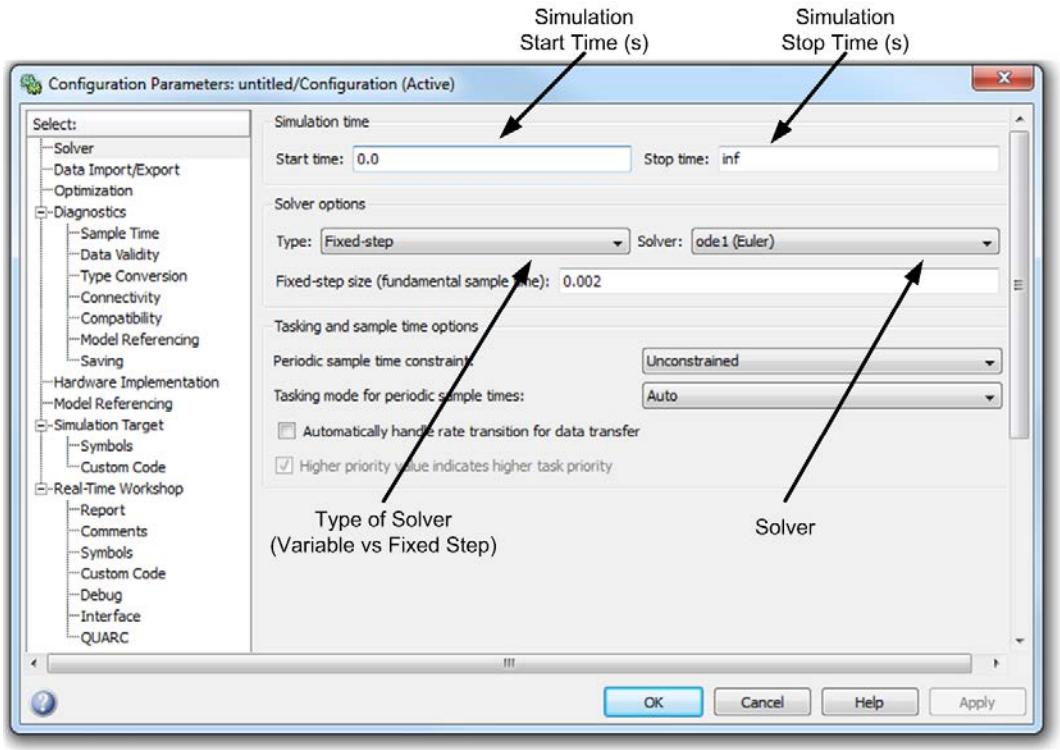


Figure 2.11: Simulation Configuration Parameters

There are several configuration settings that can be customized before running a simulation. To access these settings choose "Configuration Parameters" from the "Simulation" menu. For most simulations, the *Solver* settings are all that should need to be modified. The solver settings are outlined below:

1. **Simulation Time:** The *Start* and *Stop* time settings are defined to configure the runtime of the simulation. To create a simulation that runs until stopped, enter "inf" as the *Stop time*. The general runtime of the simulation can also be configured from the model editor toolbar.

2. **Solver Options:** The *Solver Options* section provides configuration settings for the integration and solver used in the simulation.

   Fixed-step solvers use the specified step size for solving the model. The step size can be viewed as the fundamental sample time of the system. A variable-step solver continually adjusts the integration step size within the provided bounds to maximize efficiency while maintaining the specified accuracy. Generally, the performance of a simulation is inversely proportional to the step-size and the complexity of the solver.

   (a) To choose a fixed-step solver, the most efficient method is to start by modeling the system using a variable-step solver to achieve the level of accuracy that you desire.

   (b) Next, use *ode1* to simulate the model, and compare the results with the variable-step simulation.

   (c) If the results are within your desired level of accuracy, then *ode1* is the correct solver since it is the simplest and therefore most efficient solver.

   (d) If the results of the *ode1* simulation are not accurate enough, then experiment with the more complex solvers until the most efficient solver is found that meets the accuracy requirements.

   (e) Choosing a variable-step solver can be performed in much the same manner, though if the model does not define any states, a discrete solver will be used.

Once the simulation configuration settings are determined, the simulation can be started by clicking on the *Start simulation* button on the toolbar (triangle), or by choosing ”Start” from the ”Simulation” menu. To stop the simulation click on the *Stop* button on the toolbar (square), or choose ”Stop” from the menu. The simulation can also be paused by clicking on the start button during a simulation.

2.5 Tips and Tricks

- The default orientation of all the blocks is with the input ports on the left edge of the block and the output ports on the right edge. To flip blocks, click on the block selected, then go to Format | Flip Block or press Ctrl+I.
- To splice a signal line, simply right click on the line and drag the new signal to the desired location.
- To assign values and parameters to blocks, simply double click on the block.
- Simulink includes an on-line help system with detailed documentation for all of the blocks available through the library browser. To find help for a block, select the block in the library browser and choose *Help for the selected block* from the ”Help” menu.
- To name signal lines, click on the line enter a name and press return.
- To view the values at various points in the model, show port values by selecting View | Port values | Show When Hovering/Toggle When Clicked. These values can be monitored by running a simulation until the desired point, or by pausing a simulation during execution.

2.6 Example 2 - Modeling a Toy Train

In this example we will create a continuous-time Simulink model of the system described in Example 1. The model will display the response of the train in meters, to an impulse of 100N.

Exercise 1: Create a Simulink model of the Toy Train

The simplest method to create a Simulink model of the train is to using the *Transfer Function* block.

1. To create the full model, first drag a *Transfer Function* block into a new model and enter the model of the train we created in Example 1.
2. Next, add two *Step* blocks, and combine them to output a impulse of 100 N as outlined in Section 2.3.
3. Finally, connect the impulse to the input of the system model, and the output to a *Scope*. Your model should look similar to Figure 2.12.

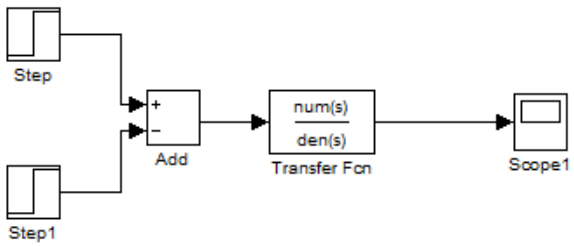


Figure 2.12: Simulink model of toy train

Exercise 2: Compare to Example 1

The response of the system should be identical to the response found in Example 1. By adding a *To Workspace* block, the output of the impulse can be sent to the workspace to compare with the impulse response of the original system. Using the **hold on** and **legend** commands, you can create the plot shown in Figure 2.13.

Exercise 3: Automate transfer function updating

To analyze the influence of the system parameters on the response, we will create a custom function to update the transfer function depending on the specified parameters. First, make sure the *Transfer Function* block references the numerator and denominator variables on the workspace (**num** and **dem**). Then, create the following MATLAB function and save it in the current directory path:

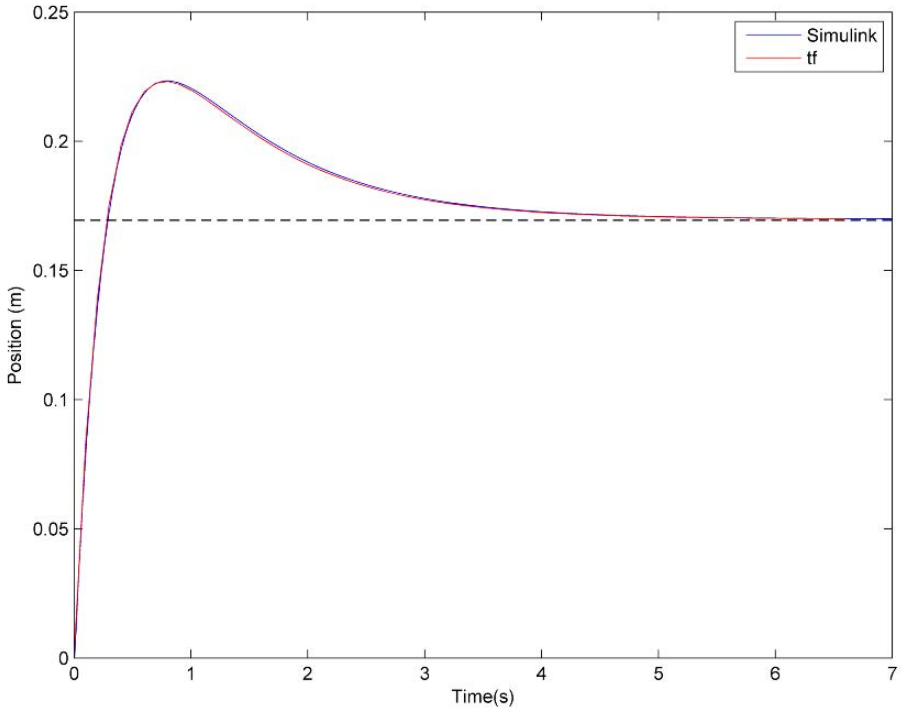


Figure 2.13: Impulse response of toy train

```
function [num,den,sys] = trainTF(m1,m2,k,mu)

c1 = mu*m1*9.81;
c2 = mu*m2*9.81;

num = [m2 c2 k];
den = [m1*m2 (m1*c2+m2*c1) (m1*k+c1*c2+ m2*k) (c1*k+c2*k) 0];
sys = tf(num,den);
end
```

The *trainTF* function essentially repeats the commands that were used to define the original transfer function, but using the specified parameters.

Exercise 4: Analyze the system response

Using the *trainTF* function, we can now create a second model with a k value of 5. Comparing the response of the updated model with the original model, we can see that the larger spring function decreases the overshoot of the response and increases the number of oscillations. The response is shown in Figure 2.14.

Create a third model with a value for m_2 of 2 kg and compare this response with the previous two systems. The mass of the second train does not effect the overshoot of the response, but decreases the resultant position since the new system has a larger inertia and rolling friction. These observations can be confirmed using a model with a k value of 5 and a value of $m_2 = 2$

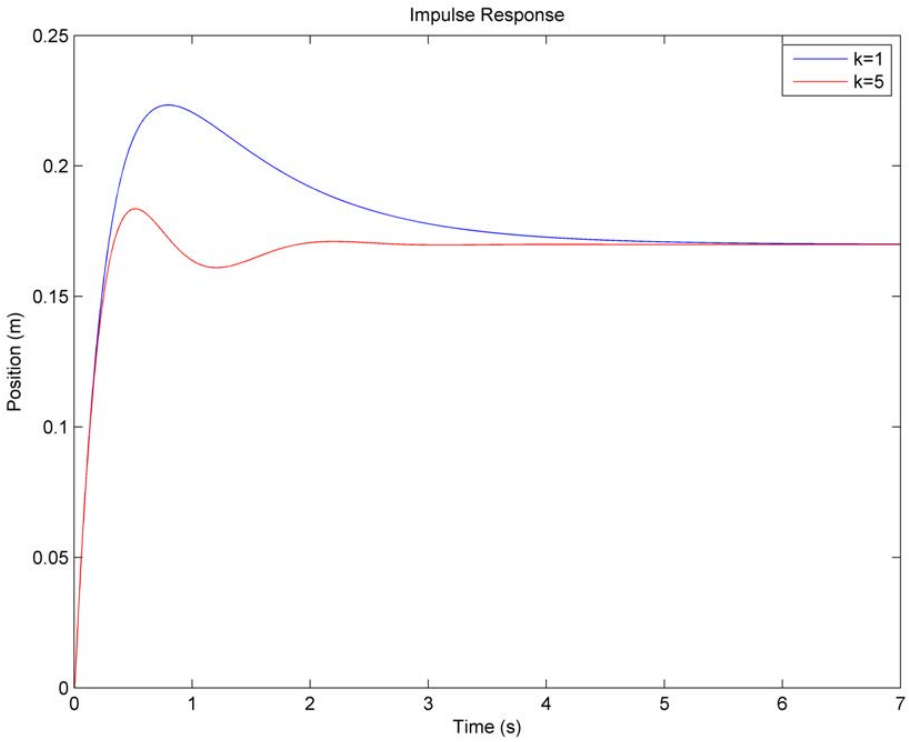


Figure 2.14: Change in impulse response with k=5

kg. The responses of all four systems are shown in Figure 2.15.

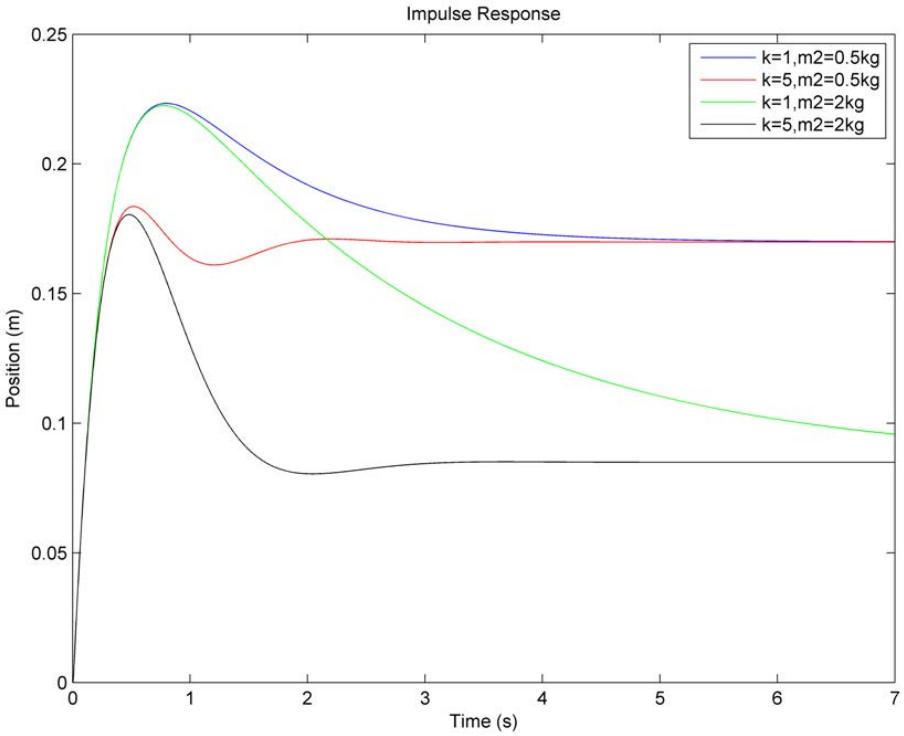


Figure 2.15: Comparison of four train responses

2.7 Example 3 - Creating an Electric Toy Train

In this example we will upgrade the toy train modeled in Example 2 to an electric train. In order to create the force necessary to drive the train, a DC motor is used. If a direct-drive configuration with a single inertial load is assumed, the first step in the upgrade would be to model the DC motor to determine how to relate the input voltage, V_m, to the output shaft speed, ω_m.

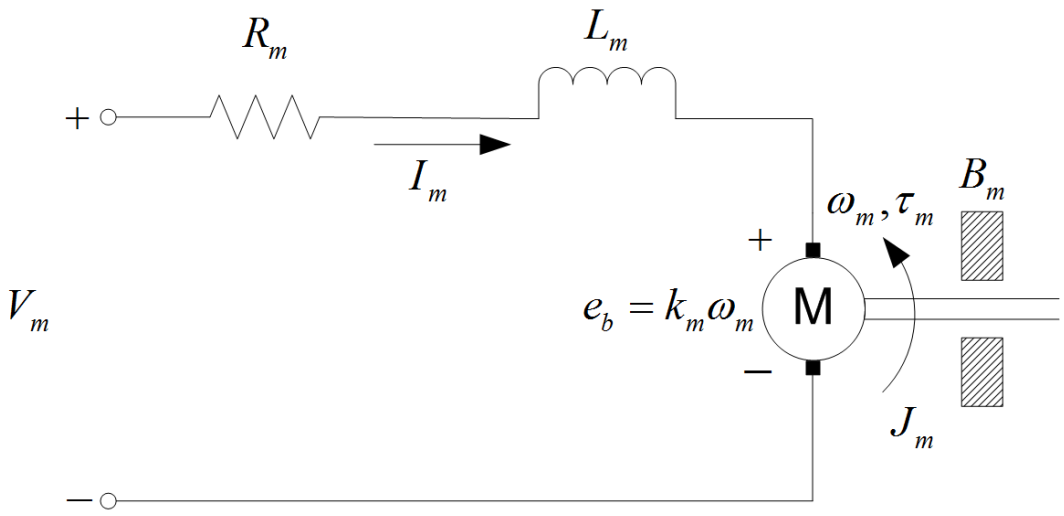


Figure 2.16: DC motor armature circuit

Since the motor model involves both electrical and mechanical components, we will begin by modeling the electrical system.

The DC motor armature circuit schematic is illustrated in Figure 2.16. R_m is the motor resistance, L_m is the inductance, and k_m is the back-emf constant.

The back-emf (electromotive) voltage, e_b, depends on the speed of the motor shaft and the back-emf constant of the motor according to:

$$e_b(t) = k_m \omega_m(t) \tag{2.10}$$

Using Kirchoff's Voltage Law which states the the voltages in a closed loop circuit must be equal to zero, we can write the following equation:

$$V_m(t) - R_m I_m(t) \frac{dI_m(t)}{dt} - k_m \omega_m(t) = 0 \tag{2.11}$$

Solving for $I_m(t)$, the motor current can be described as:

$$I_m(t) = \frac{V_m(t) - k_m \omega_m(t)}{R_m} \tag{2.12}$$

Since the motor inductance L_m is far less than the motor resistance, it can be ignored. Given the viscous friction along the motor shaft, B_m, the equation of motion becomes:

$$J_m \left(\frac{d}{dt} \omega_m(t) \right) + B_m \omega_m(t) = \tau_m(t) \tag{2.13}$$

where J_m is the moment of inertia of the load and τ_m is the total torque applied on the load.

The motor torque is proportional to the voltage applied, and is described as:

$$\tau_m(t) = \eta_m k_t I_m(t) \tag{2.14}$$

where k_t is the current-torque constant $(N.m/A)$, η_m is the motor efficiency, and I_m is the armature current.

We can express the motor torque with respect to the input voltage $V_m(t)$ and load shaft speed $\omega_l(t)$ by substituting the motor armature current given by Equation 2.12 into the current-torque relationship:

$$\tau_m(t) = \frac{\eta_m k_t \left(V_m(t) - k_m \omega_m(t)\right)}{R_m} \tag{2.15}$$

Now that we have the motor torque expressed as a function of the input voltage, we substitute Equation 2.15 into the equation of motion Equation 2.13:

$$J_m \left(\frac{d}{dt}\omega_m(t)\right) + B_m \omega_m(t) = \frac{\eta_m k_t \left(V_m(t) - k_m \omega_m(t)\right)}{R_m} \tag{2.16}$$

Exercise 1: Create a Simulink model

To create a motor model in the time domain that relates the motor voltage, V_m, to the shaft speed,ω_m, the first step is to rearrange Equation 2.16. The resultant equation is:

$$\frac{d}{dt} w_m(t) = \frac{(\frac{\eta_m k_t}{R_m}) V_m(t) - (\frac{\eta_m k_t k_m + B_m R_m)}{R_m}) \omega_m(t)}{J_m} \tag{2.17}$$

The Simulink model of Equation 2.17 can be built by translating each operation in the equation to an equivalent Simulink block. For example, the first operation in the expression is to multiply the motor voltage by the actuator gain. In Simulink this is commonly done using a gain. The resultant system is shown in Figure 2.17.

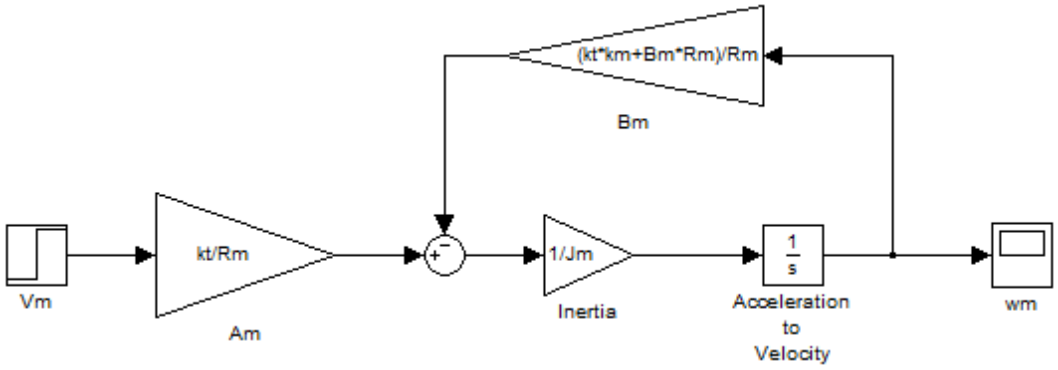


Figure 2.17: Motor model

Exercise 2: Plot the system response

Enter the following values for the system parameters:

- k_t = 0.0274 Nm/Amp
- k_m = 0.0274 Volt-sec
- R_m = 4 Ω
- J_m = 3.23 $\times$ 10^{-6} kgm^2/s^2
- B_m = 3.5 $\times$ 10^{-6} Nms

The step response of the system to a 9V step should be sharp rise in the shaft velocity until the shaft reaches its peak velocity of around 322 rad/s, shown in Figure 2.18.

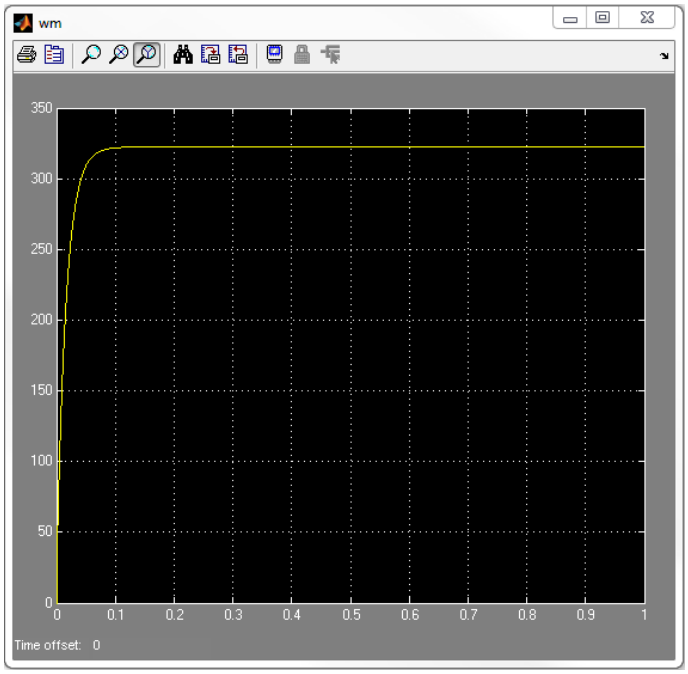


Figure 2.18: Step response of DC Motor

Exercise 3: Connect the motor to the train

To add the motor model to the Simulink model from Section 1.6, use the following procedure:

1. Add a *Subsystem* block to the motor model.
2. Move all of the elements of the motor model except for the *Step* and *Scope* block to the subsystem.
3. Rename the input and output from the subsystem block. Connect the *Step* and *Scope* to the new subsystem and test the model to ensure that it is still functional. Your motor model should now resemble Figure 2.19.

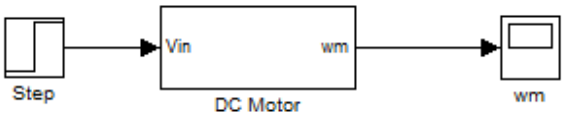


Figure 2.19: Motor model subsystem

4. Open the Simulink model from Section 1.6.
5. Copy all of the elements of the motor model into the train model.

Now that the motor model has been added into the train model, a number of modifications need to be made to the motor model to connect it to the train shown in Figure 2.20. These changes are outlined below:

1. The first step in connecting the DC motor to the train is to convert the output from the motor model from the motor angular velocity, ω_m, to the angular acceleration of the motor shaft, α_m. To make this change simply remove integrator block. Don't forget to rename the output block.
2. Next add an input into the subsystem that accepts the angular velocity feedback, since the angular velocity feedback is dependent on the overall system position. Your motor model subsystem should now resemble Figure 2.20.

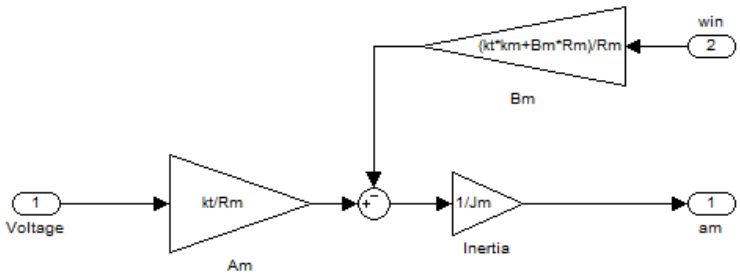


Figure 2.20: Modified motor model subsystem

3. To connect the angular acceleration output from the motor model to the train transfer function from Section 1.6, the angular acceleration of the motor shaft must be translated into an applied force onto the train. The angular acceleration must first be converted into a load torque by applying Newton's Second Law. If we assume that the motor is connected to a wooden train wheel 40mm in diameter and 5mm thick, the inertia of the wheel, J_l, is roughly 1.34×10^{-005}. If we ignore slipping, the force applied to the train can therefore be expressed as:

$$F = \frac{J_l \alpha_m}{r} \tag{2.18}$$

where r is the radius of the wheel. To complete the model, add two additional gain blocks to the model and connect the angular acceleration output from the motor to the inertia gain, then to the inverse of the radius and finally to the input to the train model.

4. To complete the motor feedback loop, you must convert the position of the train to the angular velocity of the motor. This can be done by simply adding a gain to convert the position of the train into the resultant position of the wheel, and a *Derivative* block to convert the wheel position into angular velocity. The final model is shown in Figure 2.21

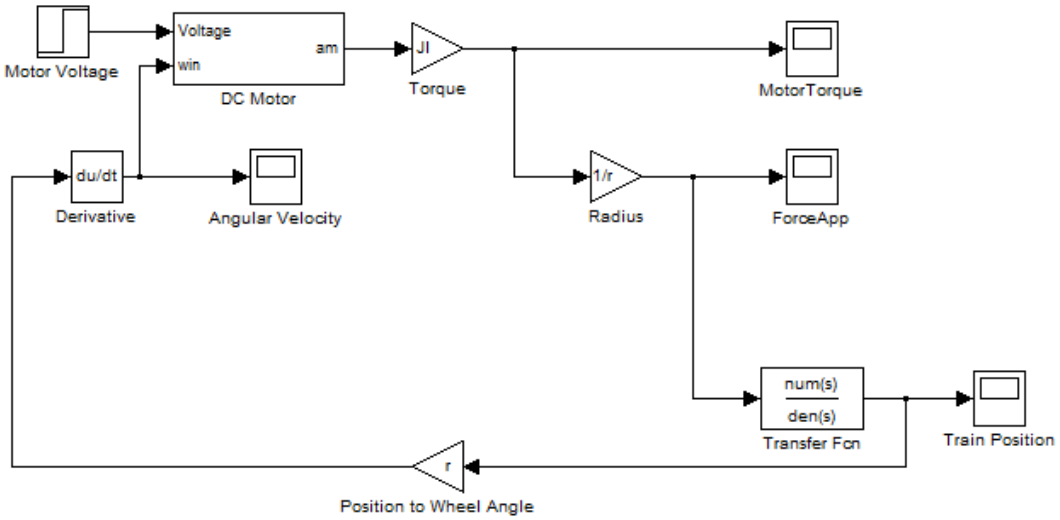


Figure 2.21: DC motor powered train model

The position step response of the overall system should resemble Figure 2.22, and the torque response of the motor should resemble Figure 2.23.

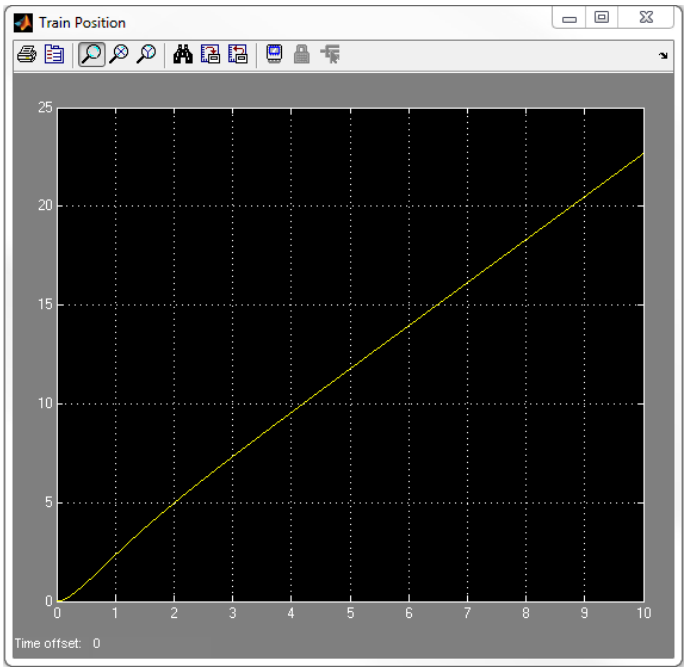


Figure 2.22: Position response of motor powered train model

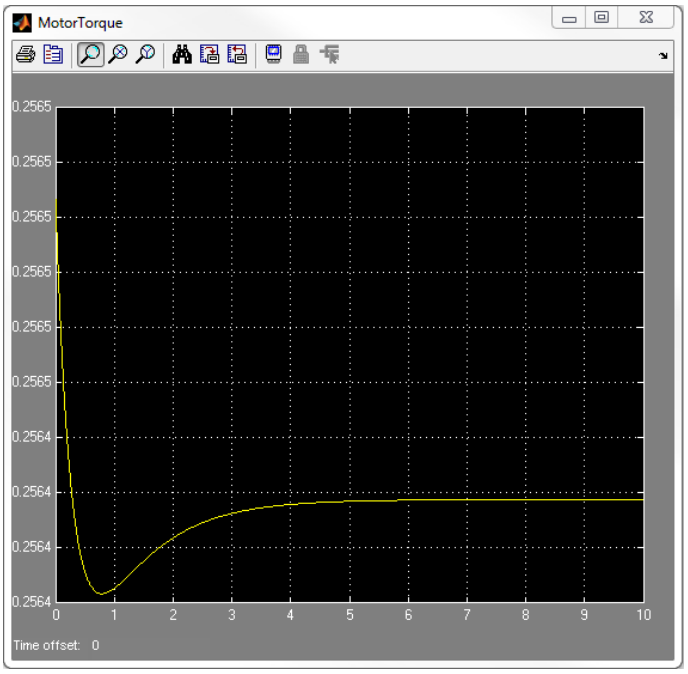


Figure 2.23: Torque response of motor powered train model

3 QUARC

3.1 Introduction

QUARC is Quanser's rapid prototyping and production system for real-time control. QUARC integrates seamlessly with Simulink to allow Simulink models to be run in real-time on a variety of targets. In essence, QUARC facilitates the creation of Simulink models that are able to run in real-time on and off the PC.

3.2 Getting Started

QUARC provides several libraries of custom blocks for building real-time models including several custom sources, sinks, and generic blocks for building system models.

Section 3.3 provides an overview of the configuration process. When the model settings are configured, the next step is to build the model and connect to the target. The final step is to connect to the real-time code to initialize the model for execution. These steps are described in more detail in Section 3.4.

The QUARC library includes several custom blocks for interfacing with external hardware. These blocks are described in more detail in Section 3.5. For more information on these and other QUARC topics see the QUARC section of the MATLAB help. Help for specific blocks can be accessed by right-clicking on the block and choosing *Help*.

3.3 Configuring a Model

The two parameters that are crucial for building and running models using QUARC are the *Solver options* and the *System target file*. The configuration parameters can be accessed through either the Configuration Parameters dialog located under the Simulation menu, or the Model Explorer.

The configuration of the *Start time*, *Stop time*, *Step size*, and *Solver* are similar to when running a conventional simulation. It is important when configuring the solver for use on an external target to ensure that the sampling time is compatible with the target machine.

The *Tasking mode for periodic sample times* parameter specifies the way periodic sample times should work.

- In Auto mode, QUARC decides whether the model should be handled using single-tasking or multi-tasking mode based on the number of sample times in the model.
- In single tasking mode, models with one sample time run in one thread with no restrictions.

The *System target file* specifies the target file that is used for code generation. To configure the target type, click on the "Browse" button which is located to the right of the *System target file* field, and choose a target type among the list of target files. The target only needs to be configured if different from the default target. The configuration parameters are saved with the model, so the system target file only needs to be configured once for each model.

3.4 Building and Running a Model

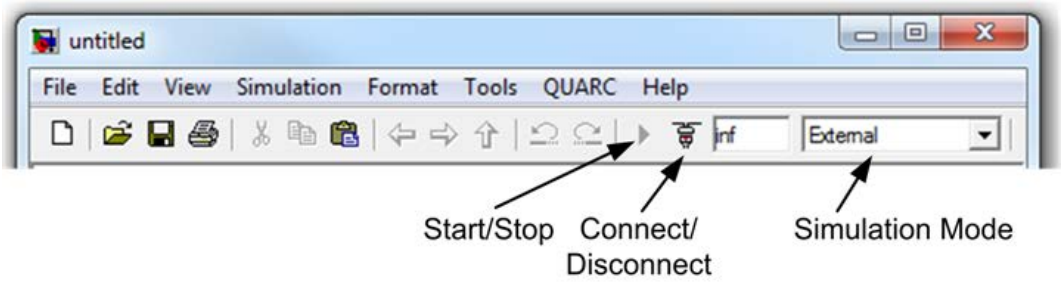


Figure 3.24: QUARC menu buttons

3.4.1 Building a Model

The next step after configuring the parameters discussed in the previous section is to build the model. This step is only necessary if the model is to run on a target in real-time. For simulating the model, start the model as discussed in Section 2.4.

The simplest method of building a model is to press Ctrl+B when the model window is active.

3.4.2 External Mode vs. Normal Simulation

In *normal* simulation mode, Simulink simulates the blocks in the model as described in Section 2.4. The simulation does not run in real-time, but executes as a part of MATLAB as quickly as possible.

When in *external* mode, real-time code is generated from the model using QUARC and runs independently of MATLAB. The host and target may be the same machine, depending on the target type for which the real-time code was generated.

There are two methods for switching back and forth between normal simulation and external mode. The easiest method is to use the toolbar of the model window to choose between the *Normal* and *External* options shown in Figure 3.24.

3.4.3 Connecting to a Model

If *External* mode is chosen, in order to run the model on the specified target machine you have to connect to the model's real-time code. The keyboard shortcut for connecting to real-time code is Ctrl+T.

3.4.4 Starting and Stopping a Model

The final step to running a model in real-time is to start the code. Starting and stopping the model is similar to starting a normal simulation. You can use the *Start real-time code* button from the

toolbar. Similarly, to stop the model you can use the *Stop real-time code* button on the toolbar, or you can press the *Pause* key.

3.5 Accessing Hardware

QUARC supports a variety of data acquisition cards, including Quanser's own Q4 and Q8 Hardware-in-the-Loop (HIL) boards, the National Instruments PCI-6259 and many more. Hardware connected to these boards can be accessed by simply placing the appropriate blocks in your Simulink diagram. The blocks are found in the library browser under QUARC Targets | Data Acquisition | Generic and require very little additional configuration. Please consult the *Using Hardware* QUARC demos for more examples of the HIL blocks.

3.5.1 HIL Initialize Block

This block can be found in the Simulink Library Browser under the QUARC Targets | Data Acquisition | Generic | Configuration and should be placed in all models that require hardware access of some type. The HIL Initialize block associates a board name with a particular HIL board. The name assigned to each board will appear in the list of boards for every other HIL block in the diagram.

An important function of the HIL Initialize block is to configure the I/O on the card. For example, many cards allow the digital I/O lines to be programmed as inputs or outputs. The *Digital Inputs* and *Digital Outputs* tabs are used to configure which digital I/O lines are inputs and outputs respectively. Similarly, many cards permit the range of the analog inputs and outputs to be programmed. The *Analog Inputs* and *Analog Outputs* tabs are used for this purpose. Many other features of data acquisition cards are configured using the tabs of the HIL Initialize block. Be sure to check the settings on each tab carefully to ensure that your card is set up properly. Please consult the HIL Initialize help page for full details.

3.5.2 Immediate I/O HIL Blocks

These blocks are found in the Simulink Library Browser under the QUARC Targets | Data Acquisition | Generic | Immediate I/O. The immediate I/O blocks read from or write to the specified channels every time the block is executed. The channels are read from or written to immediately.

3.5.3 Buffered I/O HIL Blocks

These blocks are found in the Simulink Library Browser under QUARC Targets | Data Acquisition | Generic | Buffered I/O. The buffered I/O blocks can be used to read in buffered data. This can be done by specifying the number of samples to be buffered (buffer size) in the dialog box associated with the block. The specified number of samples are then read at the given sampling rate and buffered. These values are output from the block once all the samples have been read.

3.5.4 Timebase HIL Blocks

These blocks are found in the Simulink Library Browser under QUARC Targets | Data Acquisition | Generic | Timebases. The hardware timebase blocks read from or write to the specified channels at the sampling rate of the model and act as a timebase for the model. More specifically, whenever hardware is accessed via the Timebase HIL blocks, acquisition timing and the triggering of the model execution are done by a hardware timer on the DAQ (for better performance) rather than the target OS (e.g. Windows) system timebase.

3.6 Example 4 - Position Controlled Toy Train

In this example we will create a simple Proportional Derivative (PD) controller to improve the rise-time and overshoot of the train model created in Example 2.

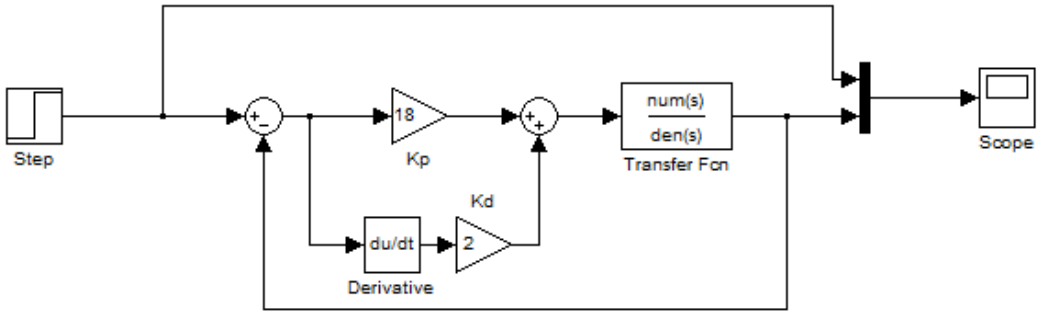


Figure 3.25: Train position controller

Exercise 1: Create a PD controller

Add the PD position controller shown in Figure 3.25 to the model train. The feedback loop can be created using a *Sum* block that subtracts the current position of the train from the command to the system.

Exercise 2: Tune the Controller

Wire both the input and the output of the system into a *Scope*, or output their data to the desktop, and plot the step-response of the system as shown in Figure 3.25. Tune K_p until the rise-time is less-then 1 second, and then increase K_d to minimize the overshoot of the system.

Exercise 3: Run the model using QUARC

Add a *System Timebase* block and run the model. Notice how the simulation is no longer running as fast as possible, but now runs in pseudo real-time.

Exercise 4: Take input from system mouse

In this exercise we will modify the toy train model to take force input from the mouse to demonstrate the real-time capabilities of QUARC.

Start by adding the following blocks to the model:

1. Add a *Host Initialize* block. This block will initilize the peripherals.

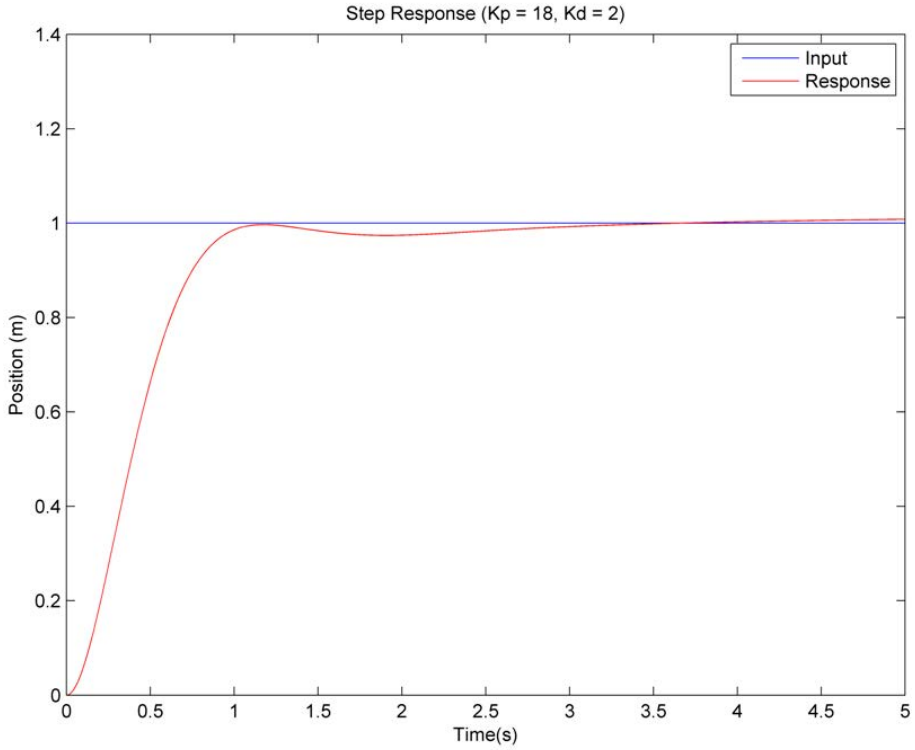


Figure 3.26: Train step response

2. Add a *Host Mouse* block from the *Host* library located in QUARC Targets | Devices | Peripherals | Host. This block will interface with the system mouse.
3. From the same library, add a *Host Keyboard* block. This blocks will access the status of a key on the keyboard.
4. Add a *Switch* from the *Commonly Used Blocks* library.
5. Add a *Signal Generator* from the *Sources* library.

Wire the new blocks as shown in Figure 3.27.

Configure the new blocks as follows:

1. Open the *Host Initialize* block properties and note the name of the host.
2. In the *Host Keyboard* block properties, choose a key to serve as the switch input (e.g. F1).
3. Open the *Host Mouse* block properties and enter the host name from the drop menu.
4. Configure the *Signal Generator* block to output a square-wave with an amplitude of 1 and frequency of 0.5.

When you run the system model, the scope should show the response of the system to the pulse train output by the *Signal Generator*. When the key specified in the *Host Keyboard* is pressed, the system should match the position of the mouse y-axis in real-time. Play with the mouse position and observe the system response.

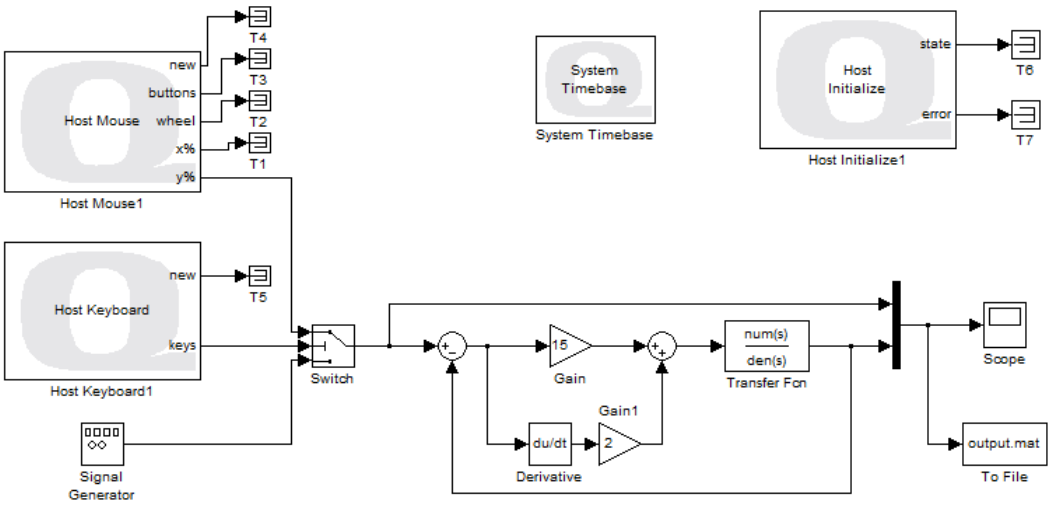


Figure 3.27: Closed-loop train model with mouse input.

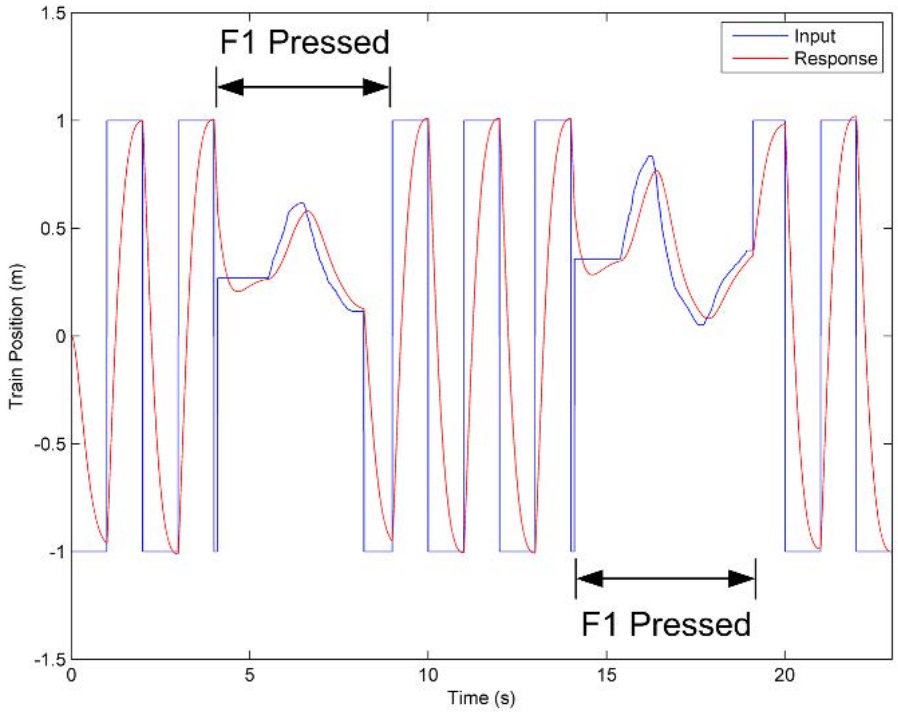


Figure 3.28: Mouse response of the toy train

4 MATLAB Cheat Sheet

& : AND
|| : OR
: NOT
= : Not equal
< : Less than
<= : Less than or equal
== : Equal
> : Greater than
>= : Greater than or equal
abs: Absolute value
angle: Returns phase angle(s) of complex number(s)
axis: Set the scales of the current plot
'b': blue
bode: Draw a Bode plot
'c': cyan
c2d: Convert continuous to discrete model
ceil: Round to the nearest integer toward ∞
char: Creates character array
clf: Clear figure
close all: Close all figures
close: Close current figure
conj: Complex conjugate
conv: Convolution
cross: Cross product
ctrb: Controllability matrix
deconv: Deconvolution
det: Determinant
dlqr: Linear-Quadratic Requlator for discrete-time system
dlsim: Simulation of discrete-time linear system
dot: Dot product
dstep: Step response of discrete-time linear system
eig: Eigenvalues of a matrix
exp: Exponential
eye: Identity matrix
feedback: Feedback interconnection of LTI systems
figure: Create a new figure
find: Finds indices of specific values
findstr: Find a string
floor: Round to the nearest integer toward -∞
'g': green
get: Query graphics object properties
hold: Hold current plot
imag: Imaginary part(s) of complex number(s)
impulse: Impulse response of continuous-time linear system
input: Prompt for user input
inv: Matrix inverse
isempty: True if matrix is empty
'k' : black
legend: Graph legend
linspace: Returns a linearly spaced vector
log: Natural logarithm
log10: Common (base-10) logarithm
loglog: Plot using log-log scale
logspace: Return a logarithmically spaced vector
lqr: Linear-Quadratic Regulator for continuous system
lsim: Simulate LTI model response
'm' : Magenta
mag2db: Convert magnitude to decibels (dB)
margin: Find gain margin, phase margin, and crossover frequencies
mean: Average
median: Median
norm: Norm of a vector
nyquist: Draw a Nyquist plot
obsv: The observability matrix
ones: Create a vector or matrix of ones
place: Compute gains to place poles of $\dot{x} = Ax + bu$
plot: Draw a plot
poly: Find characteristic polynomial
polyadd: Add two polynomials
polyfit: Fit a polynomial to data
polyval: Polynomial evaluation
'r' : Red
rand: Generate random number between 0 and 1
rank: Find the number of linearly independent rows or columns of a matrix
real: Real part of a complex number
rlocus: Draw root locus
roots: Find the roots of a polynomial
round: Round toward the nearest integer.
series: Series interconnection of LTI systems
set: Set graphics object properties
size: Dimension of a vector or matrix
sort: Sorts columns
sqrt: Square root
ss: Create state-space model
ss2tf: State-space to transfer function
ss2zp: State-space to pole-zero
std: Calculate the standard deviation
step: Plot step response
strcmp: Compare strings
subplot: Create multiple plots in figure
sum: Sum columns
tf: Create transfer function
tf2ss: Convert transfer function to state-space
tf2zp: Convert transfer function to Pole-zero
title: Add a title to current plot
'w' : White
wbw: Bandwidth frequency given the damping ratio and the rise or settling time
xlabel: Add label to x-axis
'y' : Yellow
ylabel: Add text label to y-axis
zeros: Create a vector or matrix of zeros
zp2ss: Pole-zero to state-space
zp2tf: Pole-zero to transfer function

Appendix A

Advanced Figure Commands and Properties

Some more advanced commands and functions for creating and customizing plots and other graphics are listed below.

- **subplot:** The *subplot* function is used to create multiple tiled plots in the same figure. The subplot function is called before each.
- **annotation:** The *annotation* function can be used to create a wide range of custom graphics on a plot. For example, the command

```
» annotation('textbox',[50,50,100,50],'string','Look at this!');
```

  will create a textbox that reads "Look at this!" at the location (50,50), 100px in width and 50px in height.
- **get/set:** The *get* and *set* commands are used to retrieve or change the properties of graphics objects. Graphics objects are the elements of a figure that makeup the graphics that are displayed and include figures, plots, annotations, etc. The first parameter of the *get* and *set* functions is a "handle" for that particular object. This "handle" is usually returned when the graphics element is created. The other parameters specify the properties to retrieve or update. These properties can also usually be specified when the object is created. The specific properties of various elements are listed on their respective help pages. These commands are very useful for customizing graphs and plots (colours, locations, etc.) after the objects are created. For example, to change the background colour of a figure to blue ('b'), the following commands could be used

```
» h = figure;
» set(h,'Color','b');
```

- **delete:** The *delete* function can be used to delete graphics objects. For example, if you want to delete a plot but keep the title intact

```
» figure;
» h = plot(t,y);
» title('I Will Remain!');
» delete(h);
```

- **drawnow:** The *drawnow* command is an essential command when you are updating the data shown in a plot during program execution. The command updates the current figure with any changes that are pending. If more advanced functionality is used to detect events such as a mouse click, the command also calls all pending interrupts.

Appendix B

Additional Programming Concepts

B.0.1 Data Structures

The MATLAB environment includes several classes, or data types, to accommodate various data types or storage requirements. The more basic numeric classes have already been addressed and include boolean values, numeric arrays, character arrays, and function handles. The other type of class that is available in MATLAB is referred to as a heterogeneous container, meaning it can hold a variety of data types together. There are two types of containers: structures and cell arrays.

- **Structures**
  Structures in MATLAB provide a means to store different types of data together in a single entity. Structures consist of data arrays and *fields*, which serve as a label for the individual datasets. Within each field, data arrays can be any type and valid array dimension. Structures not only facilitate more organized data storage, but also enable users to pass multiple datasets to and from functions.
  To declare a structure the command **struct** is invoked, followed by the contents of the structure. For example, the following statement creates a structure containing a scalar, a numeric array, and a character array:

```
» data = struct('Name','John','Age',32,'Times',[10.56,9.42,8.35,11.55]);
```

  To access the contents of the structure, the period "." is used to "index" into the structure as follows

```
» data.Name
```

  Output:

```
ans =
    John
```

  Arrays of structures can also be created by assigning the last index in an array to the structure. The array elements are automatically populated with the "default" fields used in the declaration. For example, to create an array of the structures declared earlier you could enter

```
» dataset(5) = data
```

Output:

```
dataset =

1x5 struct array with fields:
    Name
    Age
    Times
```

Structures can also be nested within one another by declaring a structure, and then including it in a field of a parent structure. For example, the following statement includes the structure ”student” in the parent structure ”classroom”

```
» student = struct('Name','Arya','Age','13','Marks',[87,92,88,78,81]);

» classroom = struct('Number',201,'Teacher','Mrs.  Stark','Students',student);
```

Output:

```
» classroom =

    Number:  201
    Teacher:  'Mrs.  Stark'
    Students:  [1x1 struct]

» classroom.Student

ans =

    Name:  'Arya'
    Age:  '13'
    Marks:  [87 92 88 78 81]
```

- **Cell Arrays**
  A cell array is an array of containers called *cells* in which you can store different types of data. A cell array is similar to a structure in that it contains different datatypes divided into separate containers, but uses a traditional array structure rather than fields to arrange the data. Cell arrays are created in much the same manner as creating any other array in MATLAB, but using curly braces `{ }`, instead of square. For example to create a cell array of character and numeric arrays you might use the following statements

```
» car ={'Aston Martin','DB9',[469,443,3880,4.6,4.8,190];}
```

Output:

```
car =
    'Aston Martin'    'DB9'    [1x6 double]
```

Accessing the cells within the array can be done with either the usual round or curly braces

```
» car{1,2}
```

Outputs:

```
ans =
    'DB9'
```

B.0.2 Debugging

The editor provides tools that can be used to step through and monitor code execution to help find diagnose problems. Before you begin debugging, you have to specify "breakpoints", or places where execution is paused. To specify a breakpoint click on the dash that appears on the right side of the line number, shown in Figure B.29. A small red circle will appear to denote the breakpoint position. Repeat this procedure to remove the breakpoint. To begin debugging click on the *Run* button (Green Triangle) on the toolbar, or run the program from the Desktop.

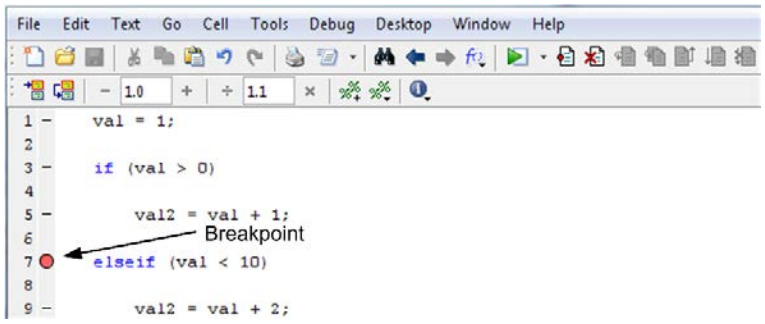


Figure B.29: Creating a breakpoint.

When the program execution is paused at a breakpoint, the position of the program execution is indicated by a small green arrow. You can view the values of variables by holding the cursor over their name anywhere in the code. To continue to step through the code, click on the *Step* button on the toolbar. To step into another function or script you can click on the *Step in* button. To continue execution click on the *Continue* button, and to halt debugging click on the *Exit debug mode* button. An example of debugging is shown in Figure B.30.

B.0.3 Tips and Tricks

Sometimes when writing complex or time critical programs, the efficiency of a program is very important. The following factors should be considered when trying to make programs more efficient:

- Though scripts are often simpler to create than functions, functions often execute faster.
- When filling in array elements in a loop, preallocate the elements of the array using the **zeros** function. This will have a significant impact on the execution speed because as the array is growing inside the loop, each time the loop iterates MATLAB must recreate the entire array.
- **While** loops are sometimes faster than **for** loops.
- Avoid using **for** loops to perform mathematical operations that can be executed using matrix algebra or the colon operator. For example, the statement

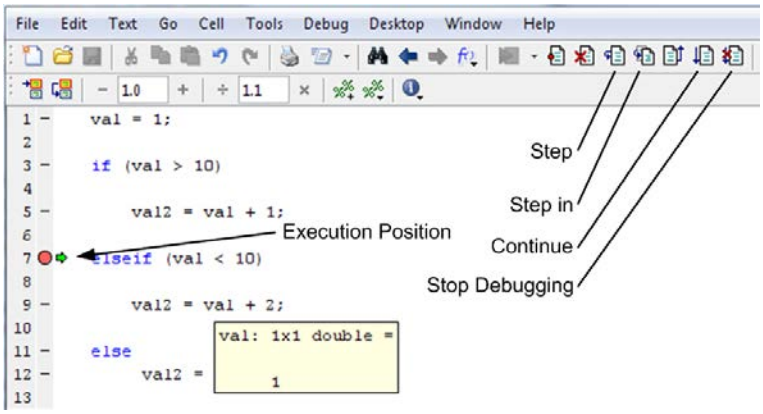


Figure B.30: Debugging a program.

```
t = 0:1:100;
y = t^2+2*t+1;
```

is much more efficient than

```
for t=0:100
    y(t) = t^2+2*t+1;
end
```

- Consider using the "short-circuiting" logical operators `&&` and `||` when possible. These operators avoid evaluating the entire condition if the first statement satisfies the condition. For example, the statement if(var > 1 `&&` var2 < 10) will continue execution if var <= 1 without checking the value of var2.

Some other tricks include:

- The key sequence **Ctrl+C** will halt execution of any running functions or scripts.
- Pressing **Tab** when entering a command into the *Command Window* will auto-complete the statement with a list of probable commands.
- Pressing the *UP* arrow at the command prompt will scroll through the previously entered commands.